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Gravitational effects on measurements of the muon dipole moments

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Abstract

If the technology for muon storage rings one day permits sensitivity to precession at the order of 10^{-8} Hz, the local gravitational field of Earth can be a dominant contribution to the precession of the muon, which, if ignored, can fake the signal for a nonzero muon electric dipole moment (EDM). Specifically, the effects of Earth's gravity on the motion of a muon's spin is indistinguishable from it having a nonzero EDM of magnitude $d_\mu \sim 10^{-29}$ e cm in a storage ring with vertical magnetic field of ~1 T, which is significantly larger than the expected upper limit in the Standard Model, $d_\mu \lesssim 10^{-36}$ e cm. As a corollary, measurements of Earth's local gravitational field using stored muons would be a unique test to distinguish classical gravity from general relativity with a bonafide quantum mechanical entity, i.e., an elementary particle's spin.

While colliders are suitable for producing heavy particles, uncovering precise detail of the underlying theory is left to precision measurements performed by dedicated experiments. This is particularly true for the electromagnetic interaction. For example, the interaction between a charged lepton ℓ and an external electromagnetic field is

$$\left\langle \ell(p+q) \middle| J^\alpha_{\mathrm{EM}}(q^2) \middle| \ell(p) \right\rangle \\ \propto \bar{u}(p+q)\left[\gamma^\alpha F_1(q^2) + \frac{i\sigma^{\alpha\beta} q_\beta}{2m_\ell} F_2(q^2) - \sigma^{\alpha\beta} q_\beta \gamma^5 F_3(q^2)\right] u(p), \tag{1}$$

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where m_ℓ is the mass of the lepton and $\sigma^{\alpha\beta} \equiv \frac{i}{2}[\gamma^\alpha, \gamma^\beta]$. When $q^2 = 0$, $F_1(0) \equiv 1$, and the other form factors have values $F_2(0) = a_\ell$, $F_3(0) = d_\ell/e$, where a_ℓ is the anomalous magnetic dipole moment, and d_ℓ is the electric dipole moment (EDM).[1] The definition of $F_1(q^2 = 0)$ is a requirement that the theory behaves like classical electrodynamics at low energies, and the values of F_2 and F_3 are purely quantum mechanical in origin and can be calculated in perturbation theory. Colliders can measure the gross scattering probability between charged particles, but they are generally insensitive to these higher-order effects. Dedicated experiments to measure the dynamics of a charged lepton's spin in external electromagnetic fields, on the other hand, can be singular opportunities to discover deviations from the Standard Model (SM), since theoretical calculations of charged-lepton dipole moments include all physics in the SM and, in principle, beyond.

Measurements of a_μ, d_e, and d_μ serve as the best opportunities to discover new physics. On the other hand, while the electron is the easiest particle to control, experimentally, the potential to discover new physics with a_e is minimized, because the value of a_e is dominated by QED, and not the SM at the weak scale, due to the lightness of the electron mass [2]. Taus can be produced in appreciable amounts at collier experiments, but because of its short lifetime, neither a_τ nor d_τ have yet been measured with meaningful precision [3–5].

The muon anomalous magnetic dipole moment a_μ was best measured to ~ 0.5 ppm by the $(g-2)_\mu$ experiment at BNL [6], but resulted in about a 2.7σ–3.6σ discrepancy with the value expected in the SM, depending on the details of the contributions of the strong interactions [7–12]. Whether or not this is a sign of new physics may be settled as improved experimental [13,14] and theoretical techniques [15–23] are employed.

The SM predictions for the values of d_e and d_μ are minuscule, due to small magnitude of CP-invariance violation in the SM and the small values of neutrino masses. The CP-invariance violation from mixing in the quark sector is the dominant contribution to charged-lepton EDMs,[2] where a nonzero muon EDM first occurs at the four loops, resulting in the upper limits $d_e \lesssim 10^{-38}$ e cm and $d_\mu \lesssim 10^{-36}$ e cm [24–26]. New physics, then, can easily play a dominant role over the SM in the contribution to the charged-lepton EDMs. The current limits on the electron and muon EDMs are $d_e < 1.6 \times 10^{-27}$ e cm (90% CL) and $d_\mu < 1.05 \times 10^{-19}$ e cm (95% CL) [27,28]. While this strong limit on d_e can imply a stronger limit on the value of d_μ than what is currently experimentally verified, the naive linear scaling in the SM $d_e/d_\mu \sim m_e/m_\mu$ can be violated in the presence of new physics [29,30]. New experimental techniques with stored muons [31] offers opportunities for experiments to improve the limit on d_μ by almost 5 orders of magnitude, i.e., down to $d_\mu \lesssim 10^{-24}$ e cm, by choosing the electromagnetic fields such that the precession due to a_μ is minimized [32,33]. While further technological advances would be required to strengthen this limit further, such improvements would be invaluable to ruling out or discovering new physics that violates CP-invariance.

A measurement of a nonzero muon EDM is often thought to be clear signal of new physics. However, this might not always be the true. There is a unique relativistic effect for particles in a cyclotron: the local gravitational field will cause the muon's spin to precess in a way that mimics the dynamics due to a bonafide dipole moment. To date, no experiment has yet verified that the quantum mechanical spin of elementary particles obey general relativity. It remains an

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[1] There can be additional terms in Eq. (1) due to toroidal moments, but their values are, in general, gauge dependent for elementary fermions [1], and will not be considered in this analysis.

[2] Even if other sources of CP-invariance violation are present due from the mixing in the leptonic sector, their contribution to d_μ is very small [24–26].