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On the time delay between ultra-relativistic particles

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ABSTRACT

The time delay between the receptions of ultra-relativistic particles emitted simultaneously is a useful observable for both fundamental physics and cosmology. The expression of the delay when the particles travel through an arbitrary spacetime has been derived recently by Fanizza et al., using a particular coordinate system and self-consistent assumptions. The present article shows that this formula enjoys a simple physical interpretation: the relative velocity between two ultra-relativistic particles is constant. This result reveals an interesting kinematical property of general relativity, namely that the tidal forces experienced by ultra-relativistic particles in the direction of their motion are much smaller than those experienced orthogonally to their motion.

1. Introduction

The time delay between the reception of ultra-relativistic (UR) particles has historically been proposed in 1968 as an astrophysical observable to measure the mass of the electronic neutrino [1]. This method has notably been applied to the delay between photons and neutrinos emitted during the supernova explosion SN1987A, yielding the upper limit $m_\nu \leq 16$ eV for the mass of the electronic neutrino (see e.g. ref. [2] and references therein). Although much less constraining than today's limits on neutrino masses obtained by the *Planck* mission [3], the photon-neutrino delay observed with SN1987A has been nevertheless one of the main arguments against the OPERA erroneous measurement of superluminal neutrinos [4].

The idea of using time delays between UR particles as a cosmological probe is more recent [5] (see also the short review [6]). Though observational applications still have to face technical difficulties [7], the time delays between, e.g., cosmic rays and γ-ray bursts are expected to provide independent measurements of the cosmological parameters in the future. However, even with perfect sources and instruments, an irreducible uncertainty comes from the fact that the particles propagate in a locally inhomogeneous universe, and are therefore affected by gravitational phenomena. This issue was recently tackled by the authors of ref. [8], hereafter FGMV15, who derived a general expression for the time delay within an arbitrary spacetime, generalizing the formula proposed in ref. [5] for the Friedmann–Lemaître–Robertson–Walker universe.

The result of FGMV15 is the following. Consider two particles P_1, P_2 emitted at the same event S with different velocities. Since one of them is slightly slower than the other, those particles are received at different times t_1, t_2 by the observer, whose difference is

$$\Delta t \equiv t_2 - t_1 = \left(\frac{m_2^2}{2E_2^2} - \frac{m_1^2}{2E_1^2} \right) \int_{t_s}^{t_1} \frac{\mathrm{d}t}{1+z(t)}, \tag{1}$$

at lowest order in the inverse of the gamma factors $\gamma_i \equiv E_i/m_i \gg 1$, where m_i and E_i are respectively the rest mass and the energy of P_i, as measured at reception in the observer's frame. In eq. (1), the redshift z is not necessarily cosmological, because the formula is valid for any geometry, but it rather relies on an arbitrary $3+1$ foliation of spacetime such that the coordinate t coincides with the observer's proper time. The integral over t must be understood as an integral along the worldline of P_1, which is approximately a null geodesic. It is remarkable that eq. (1) has exactly the same form as in a strictly homogeneous and isotropic universe.

In FGMV15, this result was derived using the geodesic-light-cone formalism [9,10], and relying on a self-consistent ansatz. However, as mentioned by the authors themselves, the simplicity of eq. (1) suggests the existence of a more general derivation, which is precisely the purpose of the present paper. In sec. 2, I show that the time-delay formula is actually equivalent to assuming that the relative velocity of two UR particles is constant during their travel. I then physically justify this surprising assump-

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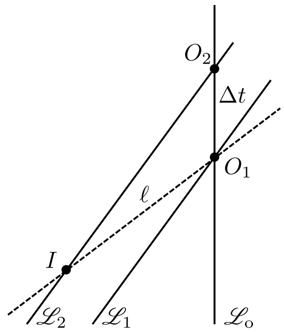

Fig. 1. Relation between (i) the time delay Δt of two UR particles following the worldlines $\mathcal{L}_1$, $\mathcal{L}_2$, in the frame of the observer $\mathcal{L}_o$; and (ii) the proper distance ℓ between those particles in the frame of the fastest one when it is detected by the observer. Solid lines indicate worldlines, while the dashed line represents the set of all events which are simultaneous with O_1 in P_1's frame.

tion in sec. 3, which reveals a general mechanism about how UR particles experience tidal forces.

2. Physical interpretation of the time-delay formula

Equation (1) has the advantages of directly involving observable quantities, and exhibits a dependence in the cosmological parameters via z. However, its physical meaning is hidden and therefore requires some reformulation. First notice that the prefactor of the integral corresponds to the difference between the velocities v_i of the particles as measured by the observer:

$$v_1 - v_2 = \sqrt{1-\frac{1}{\gamma_1^2}} - \sqrt{1-\frac{1}{\gamma_2^2}} \tag{2}$$

$$= \frac{1}{2\gamma_2^2} - \frac{1}{2\gamma_1^2} + \mathcal{O}(\gamma^{-4}). \tag{3}$$

Second, the integral of eq. (1) is proportional to the proper travel time τ_1 of P_1 from its emission at S to its observation at O_1. Indeed, since the particle is UR the evolution of its energy is essentially encoded in the lightlike redshift z as

$$1+z(t) \approx \frac{E(t)}{E_o} = \frac{\gamma(t)}{\gamma_o} = \frac{1}{\gamma_o}\frac{\mathrm{d}t}{\mathrm{d}\tau}, \tag{4}$$

where a subscript o indicates the observed value of a quantity, and τ denotes the proper time of P_1; whence

$$\int_{t_s}^{t_1} \frac{\mathrm{d}t}{1+z(t)} \approx \gamma_1 \tau_1, \tag{5}$$

so that eq. (1) takes the form

$$\Delta t = \gamma_1 (v_1 - v_2)\, \tau_1. \tag{6}$$

Though simpler than the former, the latter formula involves quantities defined in different frames, which makes it hard to interpret. The last step thus consists in translating eq. (6) into a relation between quantities in P_1's frame only. More specifically, we are going to relate the observed time delay Δt to the distance ℓ that separates the particles in P_1's frame, when P_1 is detected by the observer.

The geometry of the problem is depicted in Fig. 1. The worldline $\mathcal{L}_i$ of the particle P_i intersects the observer's worldline $\mathcal{L}_o$ at the even O_i. Define I as the event of $\mathcal{L}_2$ which is simultaneous with O_1 in the frame of P_1. This event therefore indicates the spatial position of P_2 in this frame when P_1 is detected by the observer. The spacetime separation $\Delta s^2(I, O_1)$ is therefore equal to ℓ^2.

In the following, it will be convenient to work with a Fermi normal coordinate system [11] (t, x^i) about $\mathcal{L}_o$, so that spacetime appears nearly flat in the vicinity of this worldline. As a consequence, the geodesics $\mathcal{L}_1$, $\mathcal{L}_2$ are essentially straight lines in the domain of interest. If we choose the axes of (x^i) so that $\boldsymbol{\partial}_1$ at O_1 is aligned with the direction of $O_1 E$, then the problem becomes spatially one-dimensional, and we only have to consider the coordinate $x^1 \equiv x$.

Because $I \in \mathcal{L}_2$, its coordinates (t_I, x_I) satisfy

$$x_I = v_2 (t_I - \Delta t). \tag{7}$$

Besides, a simple Lorentz transformation relates those coordinates to their counterpart $(0, -\ell)$ in P_1's frame,

$$0 = \gamma_1 (t_I - v_1 x_I) \tag{8}$$

$$-\ell = \gamma_1 (x_I - v_1 t_I). \tag{9}$$

Combining eqs. (7), (8), and (9) then yields

$$\ell = \frac{1}{\gamma_1}\frac{v_2}{1 - v_1 v_2}\Delta t, \tag{10}$$

which, once introduced in eq. (6), finally gives

$$\boxed{\ell = \frac{v_1 - v_2}{1 - v_1 v_2}\,\tau_1 = |v_{2/1}|\,\tau_1,} \tag{11}$$

where we have recognised the expression of the velocity $v_{2/1}$ of P_2 in P_1's frame [12] at the observation event. Equation (10) is equivalent to the time-delay formula (1) at lowest order in the inverse gamma factors, but its meaning is clearer: since the distance ℓ is proportional to the travel time τ_1, *the relative velocity $v_{2/1}$ of two UR particles emitted at the same event and in the same direction is constant during their travel.*

This statement—which is the physical content of the conjectures (5) in FGMV15—means that tidal forces (i.e. curvature effects) are negligible in this specific situation, even when the particles travel over cosmological distances. This is a priori surprising. Indeed, in the comparable situation of two UR particles, such as photons, emitted simultaneously but in slightly different directions, curvature is absolutely non-negligible, as it is responsible for all gravitational lensing phenomena.

In the next section, I propose a geometrical solution to this paradox, showing that for UR particles, *longitudinal* curvature is effectively much smaller than *transversal* curvature.

3. Tidal forces and ultra-relativistic particles

3.1. Geodesic deviation equation

Because the particles P_i are freely falling and very close to each other, it is reasonable to consider their worldlines $\mathcal{L}_i$ as infinitesimally separated timelike geodesics. Let us parametrize them by their own proper time $x_1^\mu(\tau)$, $x_2^\mu(\tau)$, and introduce the separation vector $\boldsymbol{\xi}$ defined by $\xi^\mu(\tau) \equiv x_2^\mu(\tau) - x_1^\mu(\tau)$. This vector is orthogonal to the geodesics, in the sense that $\xi^\mu u_\mu = 0$, where $\boldsymbol{u}$ denotes the four-velocity of one of the particles. Physically speaking, $\boldsymbol{\xi}$ represents the spatial separation of the particles in their rest frame; its norm $\xi^\mu \xi_\mu$ is thus nothing but the ℓ^2 introduced in the previous section.

The evolution of $\boldsymbol{\xi}$ with the particles' proper time τ is given by the geodesic deviation equation [11]