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Hawking radiation and the Stefan–Boltzmann law: The effective radius of the black-hole quantum atmosphere

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ABSTRACT

It has recently been suggested (S.B. Giddings (2016) [2]) that the Hawking black-hole radiation spectrum originates from an effective quantum "atmosphere' which extends well outside the black-hole horizon. In particular, comparing the Hawking radiation power of a $(3+1)$-dimensional Schwarzschild black hole of horizon radius r_{H} with the familiar Stefan–Boltzmann radiation power of a $(3+1)$-dimensional flat space perfect blackbody emitter, Giddings concluded that the source of the Hawking semi-classical black-hole radiation is a quantum region outside the Schwarzschild black-hole horizon whose effective radius r_{A} is characterized by the relation $\Delta r \equiv r_{\text{A}} - r_{\text{H}} \sim r_{\text{H}}$. It is of considerable physical interest to test the general validity of Giddings's intriguing conclusion. To this end, we study the Hawking radiation of $(D+1)$-dimensional Schwarzschild black holes. We find that the dimensionless radii $r_{\text{A}}/r_{\text{H}}$ which characterize the black-hole quantum atmospheres, as determined from the Hawking black-hole radiation power and the $(D+1)$-dimensional Stefan–Boltzmann radiation law, are a decreasing function of the number $D+1$ of spacetime dimensions. In particular, it is shown that radiating $(D+1)$-dimensional Schwarzschild black holes are characterized by the relation $(r_{\text{A}} - r_{\text{H}})/r_{\text{H}} \ll 1$ in the large $D \gg 1$ regime. Our results therefore suggest that, at least in some physical cases, the Hawking emission spectrum originates from quantum excitations very near the black-hole horizon.

1. Introduction

The Hawking evaporation process of black holes seems, at first glance, to be characterized by a non-unitary evolution of quantum fields in curved spacetimes [1]. In particular, according to Hawking's original analysis, matter fields in a *pure* quantum state may collapse to form a black hole which eventually evaporates into a *mixed* thermal state [1]. Since a unitary temporal evolution of quantum states is one of the cornerstones of quantum mechanics, it is widely believed that the semi-classical Hawking radiation spectra of evaporating black holes should be modified in order to restore quantum unitarity [2,3].

What is the characteristic lengthscale associated with these yet unknown quantum modifications? It is commonly believed that the Hawking emission spectra of evaporating black holes originate from quantum excitations in the near-horizon $\Delta r = r - r_{\text{H}} \ll r_{\text{H}}$ region [1,3]. It is therefore widely expected [3] that the required quantum modifications of the semi-classical Hawking radiation spectra would also be characterized by this relatively short [4] lengthscale $\Delta r \ll r_{\text{H}}$.

However, in a very interesting work, Giddings [2] has recently suggested that the radiation spectrum of an evaporating black hole originates from an effective quantum "atmosphere" which extends well outside the black-hole horizon. In particular, by comparing the numerically computed [5,6] Hawking radiation power P_{BH} of an evaporating $(3+1)$-dimensional Schwarzschild black hole of horizon radius r_{H} with the familiar Stefan–Boltzmann radiation power $P_{\text{BB}} = \sigma A T^4$ [7] of a $(3+1)$-dimensional flat space perfect blackbody emitter of radius r_{A}, Giddings concluded that the source of the Hawking radiation is a quantum region (the effective black-hole atmosphere) located outside the black-hole horizon and whose effective radius r_{A} is characterized by the relation

$$\Delta r \equiv r_{\text{A}} - r_{\text{H}} \sim r_{\text{H}} \; . \tag{1}$$

As emphasized in [2], the relation (1), which characterizes the $(3+1)$-dimensional Schwarzschild black hole, is consistent with the existence of an effective emitting atmosphere which extends well outside the black-hole horizon.

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It is of physical interest to test the general validity of Giddings's intriguing conclusion (1). In particular, one naturally wonders whether the relation (1), which characterizes the effective quantum atmosphere of the $(3+1)$-dimensional Schwarzschild black hole, is a generic feature of *all* evaporating black holes?

In order to address this important question, in this paper we shall study the Hawking radiation powers of $(D+1)$-dimensional Schwarzschild black holes. In particular, following [2] we shall define the effective radii $r_A(D)$ of the black-hole quantum atmospheres by equating the Hawking radiation powers of the $(D+1)$-dimensional black holes with the corresponding Stefan–Boltzmann radiation powers of flat space perfect blackbody emitters. Below we shall explicitly show that the dimensionless radii r_A/r_H, which characterize the effective black-hole quantum atmospheres, are a *decreasing* function of the number $D+1$ of spacetime dimensions. In particular, our results (to be presented below) suggest that radiating $(D+1)$-dimensional Schwarzschild black holes are characterized by the relation $(r_A - r_H)/r_H \ll 1$ [see Eq. (17) below] in the large $D \gg 1$ regime.

2. The Hawking radiation spectra of $(D+1)$-dimensional Schwarzschild black holes

We study the Hawking emission of massless scalar fields from $(D+1)$-dimensional Schwarzschild black holes. The semi-classical Hawking radiation power for one bosonic degree of freedom is given by the integral relation [1,6,8,9]

$$P_{BH} = \frac{\hbar}{2^{D-1}\pi^{D/2}\Gamma(D/2)} \sum_j \int_0^\infty \Gamma \frac{\omega^D}{e^{\hbar\omega/T_{BH}} - 1} d\omega \, , \tag{2}$$

where j denotes the angular harmonic indices of the emitted field modes, and

$$T_{BH} = \frac{(D-2)\hbar}{4\pi r_H} \, , \tag{3}$$

is the semi-classical Bekenstein–Hawking temperature of the black hole. Here r_H is the horizon radius of the black hole [10,11]. The dimensionless coefficients $\Gamma = \Gamma(\omega; j, D)$, which are known as the greybody factors [6] of the composed black-hole–field system, quantify the interaction of the emitted fields with the curved black-hole spacetime.

3. The effective radius of the black-hole quantum atmosphere

As pointed out by Giddings [2], one may define the effective radius of the black-hole quantum atmosphere by equating the Hawking radiation power P_{BH} of the emitting black hole with the corresponding radiation power P_{BB} of a flat space perfect blackbody emitter. The scalar radiation power of a spherically-symmetric blackbody (BB) of temperature T and radius R in $D+1$ spacetime dimensions is given by the generalized Stefan–Boltzmann radiation law [12]

$$P_{BB} = \sigma A_{D-1}(R) T^{D+1} \, , \tag{4}$$

where

$$\sigma = \frac{D\Gamma(D/2)\zeta(D+1)}{2\pi^{D/2+1}\hbar^D} \tag{5}$$

is the generalized $[(D+1)$-dimensional$]$ Stefan–Boltzmann constant and

$$A_{D-1}(R) = \frac{2\pi^{D/2}}{\Gamma(D/2)} R^{D-1} \tag{6}$$

is the surface area of the $(D+1)$-dimensional emitting body.

Following [2], we shall define the effective radius r_A of the black-hole quantum atmosphere from the relation [13,14]

$$P_{BH}(r_H, T_{BH}) = P_{BB}(r_A, T_{BH}) \, . \tag{7}$$

Taking cognizance of Eqs. (3), (4), (5), (6), and (7), one finds

$$r_A = \Big[\frac{\pi}{D\zeta(D+1)}\Big(\frac{4\pi}{D-2}\Big)^{D+1} \bar{P}_{BH}\Big]^{\frac{1}{D-1}} \times r_H \tag{8}$$

for the effective radiating radius of the $(D+1)$-dimensional Schwarzschild black hole, where

$$\bar{P}_{BH} \equiv P_{BH} \times \frac{r_H^2}{\hbar} \tag{9}$$

is the scaled Hawking radiation power of the black hole.

Our main goal is to determine the functional dependence $r_A = r_A(D)$ of the effective radius (8) of the black-hole quantum atmosphere on the spacetime dimension $D+1$ of the radiating black hole.

4. The radius of the black-hole quantum atmosphere: numerical and analytical results

In the present section we shall study the functional dependence $\bar{r}_A = \bar{r}_A(D)$ of the dimensionless ratio

$$\bar{r}_A \equiv \frac{r_A - r_H}{r_H} \tag{10}$$

which characterizes the effective quantum atmospheres of the radiating $(D+1)$-dimensional black holes.

4.1. *The $(3+1)$-dimensional case*

The Hawking radiation power of scalar quanta from a $(3+1)$-dimensional Schwarzschild black hole is given by [15]

$$P_{BH}(D=3) = 2.976 \times 10^{-4} \frac{\hbar}{r_H^2} \, . \tag{11}$$

Substituting (11) into (8), one finds

$$r_A = 2.679 \times r_H \tag{12}$$

for the effective radius of the black-hole quantum atmosphere. This relation yields

$$\bar{r}_A = 1.679 \tag{13}$$

for the dimensionless radius (10) which characterizes the effective atmosphere of the $(3+1)$-dimensional Schwarzschild black hole.

4.2. *$(D+1)$-dimensional black holes: intermediate D-values*

In the previous subsection we have seen that the effective quantum atmosphere of the $(3+1)$-dimensional Schwarzschild black hole is characterized by the relation $\bar{r}_A \equiv (r_A - r_H)/r_H = O(1)$ [see Eq. (13)]. This relation implies that, in the $(3+1)$-dimensional case, the effective quantum atmosphere of the black hole extends well outside the black-hole horizon. This finding supports the interesting conclusion presented in [2] for the $(3+1)$-dimensional case.

We shall now show that the dimensionless radii $\bar{r}_A(D)$ [see Eqs. (8) and (10)], which characterize the effective quantum atmospheres of the radiating $(D+1)$-dimensional Schwarzschild black holes, are a decreasing function of the number D of spatial dimensions.