Physics Letters B 757 (2016) 307–311

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Anti-Unruh phenomena

W.G. Brenna [a], Robert B. Mann [a], Eduardo Martín-Martínez [b,c,d,*]

[a] Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada
[b] Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada
[c] Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada
[d] Perimeter Institute for Theoretical Physics, Waterloo, Ontario, N2L 2Y5, Canada

ARTICLE INFO

Article history:
Received 16 October 2015
Accepted 1 April 2016
Available online 8 April 2016
Editor: S. Dodelson

ABSTRACT

We find that a uniformly accelerated particle detector coupled to the vacuum can cool down as its acceleration increases, due to relativistic effects. We show that in (1+1)-dimensions, a detector coupled to the scalar field vacuum for finite timescales (but long enough to satisfy the KMS condition) has a KMS temperature that decreases with acceleration, in certain regimes. This contrasts with the heating that one would expect from the Unruh effect.

1. Introduction

In 1976, it was proposed that the inequivalence of field quantization schemes associated with inertial and accelerated observers implied that observers uniformly accelerating in the Minkowski vacuum (as seen by inertial observers) would detect a thermal bath of particles [1]. Specifically, an accelerated particle detector coupled to the Minkowski vacuum would experience a thermal response [2], a phenomenon known as the Unruh effect. The temperature T of this thermal bath was found to be proportional to the magnitude a of the proper acceleration of the detector, with $T = a/2\pi$. The Unruh effect has been predicted and derived in contexts as disparate as axiomatic quantum field theory [3], via Bogoliubov transformations [2], and in studies of the response of non-inertial particle detectors both perturbatively [2] and non-perturbatively [4–7], and even for non-uniformly accelerated trajectories [8,9]. More recently non-perturbative techniques developed in [4] have been used to prove that within optical cavities in (1+1)-dimensions an accelerated detector equilibrates to a thermal state whose temperature is proportional to acceleration. This holds independently of the cavity boundary conditions, provided the detector is allowed enough interaction time [10].

Since all investigations so far have found that a particle detector coupled to the vacuum will detect more particles when it is accelerated than when undergoing inertial motion, we typically regard the Unruh effect as a universal phenomenon: simply put, 'accelerated detectors get hotter'. The common denominator in nearly all previous investigations is that the response of non-inertial detectors is studied for long interaction times, or for a field quantized in free infinite open space. However on empirical grounds, finite time studies with different boundary conditions are arguably relevant. Any experimental setup based on quantum optics (e.g. an atom accelerating through an optical cavity) will necessarily require particular boundary conditions rather than infinite space.

But do accelerated detectors always become hotter? In this paper we address this question using both perturbative and non-perturbative tools. Previous numerical work on accelerating Unruh–deWitt detectors in cavities interacting for long times found that, as expected, a detector gets hotter and its temperature is proportional to its acceleration; $T \propto a$ [10]. However, due to the finite length and time scales, the slope was not found to be $1/2\pi$. In this paper we find that when shorter interaction times comparable to the characteristic Heisenberg time of the detector are considered the transition probability of an accelerated detector can actually *decrease* with acceleration. This is possible because even an inertial detector switched on for a finite time in the ground state, and coupled to the Minkowski vacuum, will not remain completely 'cold' but will click due to switching noise and vacuum fluctuations (see [11] and [4] for a perturbative and non-perturbative analysis respectively).

One may be tempted to argue that this effect is due to transient behaviour. This suspicion may become even stronger given that the effect only manifests itself for times comparable to the atomic Heisenberg time. However, what makes our result surpris-

* Corresponding author.
E-mail addresses: wbrenna@uwaterloo.ca (W.G. Brenna), rbmann@uwaterloo.ca (R.B. Mann), emartinm@uwaterloo.ca (E. Martín-Martínez).

http://dx.doi.org/10.1016/j.physletb.2016.04.002
0370-2693/

ing is that we find no clear evidence that we should associate this behaviour with non-equilibrium transient effects, despite the short interaction time. Rather we find that the response of such detectors can be regarded as non-transient insofar as they satisfy the KMS condition, and a KMS temperature (which decreases with acceleration) can therefore be defined [12,13]. This would mean that these 'transients' are of a rather special kind that satisfy detailed balance, a condition which states that each elementary process should be equilibrated by its reverse process, and which is characteristic of equilibrium scenarios.

2. Transition probability of an accelerated detector

To model the field-detector interaction it is commonplace to use the Unruh–DeWitt (UDW) model [14], which consists of a point-like two-level quantum system that couples to a scalar field along its trajectory. We will first regard spacetime as a flat static cylinder with spatial circumference $L > 0$ (we will later consider the limit $L \to \infty$). This cylinder topology is equivalent to imposing periodic boundary conditions relevant to laboratory systems including closed optical cavities, such as optical-fibre loops [15], and superconducting circuits coupled to periodic microwave guides [16,17].

The coupling of the field to the detector is described by the UDW Hamiltonian [14]

$$H_I = \lambda\, \chi(\tau)\mu(\tau)\phi(x(\tau), t(\tau)), \tag{1}$$

where τ is the detector's proper time, $\mu(\tau) = \sigma_x(\tau) = e^{\mathrm{i}\Omega\tau}\sigma^+ + e^{-\mathrm{i}\Omega\tau}\sigma^-$ is the detector's monopole moment (with $\sigma^\pm$ being SU(2) ladder operators), and $\chi(\tau)$ is the switching function. For most of the paper we will consider $\chi(\tau)$ to be Gaussian

$$\chi(\tau) = e^{-\tau^2/2\sigma^2}, \tag{2}$$

so that σ establishes the timescale of the interaction between the field and the detector. The time evolution operator under this Hamiltonian is given by the following perturbative expansion:

$$U = \mathbb{1} + U^{(1)} + \mathcal{O}(\lambda^2) = \mathbb{1} - \mathrm{i}\int_{-\infty}^{\infty} \mathrm{d}t\, H_I(t) + \mathcal{O}(\lambda^2)$$
$$= -\mathrm{i}\lambda \sum_m (I_{+,m} a_m^\dagger \sigma^+ + I_{-,m} a_m^\dagger \sigma^- + \text{H.c.}) + \mathcal{O}(\lambda^2),$$

where the sum over m takes discrete values due to the periodic boundary conditions ($k = 2\pi m/L$). L is the scale of the natural IR cutoff (we neglect the interaction of the detector with the zero mode [18]), a_m and $a_m^\dagger$ are field mode annihilation and creation operators, and

$$I_{\pm,m} = \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{4\pi|m|}} e^{\pm \mathrm{i}\Omega\tau + \frac{2\pi\mathrm{i}}{L}(|m|t(\tau) - mx(\tau)) - \tau^2/2\sigma^2}, \tag{3}$$

which can be easily worked out from equation (1), expanding the field in plane-wave modes and substituting the expression for the monopole moment. If we consider a detector in its ground state, coupled to the vacuum state of the field, the transition probability at leading order in the perturbative expansion, will be given by

$$\mathcal{P} = \sum_{m\neq 0} |\langle 1_m, e|U^{(1)}|0, g\rangle|^2 = \lambda^2 \sum_{m\neq 0} |I_{+,m}|^2. \tag{4}$$

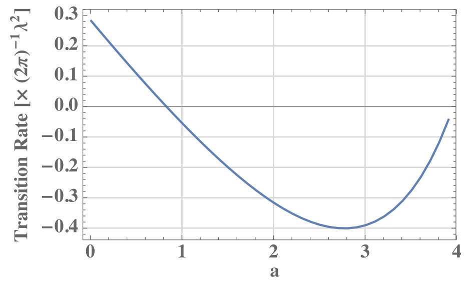

Fig. 1. Transition rate (in units of $2\pi\lambda^{-2}$) as a function of acceleration for $T = 1$, $\Omega = 2$, $L = 20$. Notice the decreasing transition rate with acceleration for low accelerations.

3. Evidence of the 'anti-Unruh' effect

For a uniformly accelerated two-level detector in a periodic cavity, the probability of transition takes the form

$$\mathcal{P} = \lambda^2 \sum_{n,\epsilon} \left| \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{4\pi n}} e^{\mathrm{i}\Omega\tau + 2\pi n \mathrm{i}\left(\frac{\epsilon}{aL}\left[e^{\epsilon a\tau} - 1\right]\right) - \tau^2/2\sigma^2} \right|^2 \tag{5}$$

upon substituting (3) into (4) and using

$$[|m|t(\tau) - mx(\tau)] = \frac{n\epsilon}{a}\left[e^{\epsilon a\tau} - 1\right], \tag{6}$$

where $m = -\epsilon n$ where $n \in \mathbb{Z}^+$, $\epsilon = \pm 1$, and $t(\tau) = a^{-1}\sinh(a\tau)$ and $x(\tau) = a^{-1}(\cosh(a\tau) - 1)$. As per our comments in the introduction, when $a \to 0$, $\mathcal{P}$ does not vanish since we are considering a finite time interaction [4,11].

Since the switching function is symmetric about $t = 0$, the overall contribution of the right-moving modes is equal to the overall contribution of the left-moving modes, so (5) simplifies to

$$\mathcal{P} = 2\lambda^2 \sum_{n>0} \left| \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{4\pi n}} e^{\mathrm{i}\Omega\tau + 2\pi n \mathrm{i}\left(\frac{1}{aL}\left[e^{a\tau} - 1\right]\right) - \tau^2/2\sigma^2} \right|^2, \tag{7}$$

which can be recast as

$$\mathcal{P} = \frac{-\lambda^2}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\tau \int_{-\infty}^{\infty} \mathrm{d}\tau' e^{\mathrm{i}\Omega(\tau-\tau') - \frac{\tau^2+\tau'^2}{2\sigma^2}} \log[1 - e^{\frac{2\pi\mathrm{i}}{aL}\left[e^{a\tau} - e^{a\tau'}\right]}] \tag{8}$$

upon summing the series in n. The first interesting feature to note in this expression is that the probability is not monotonically increasing with acceleration for all values of the parameters, contrary to expected intuition from the Unruh effect.

For illustration, before employing the Gaussian switching function, let us first compute the transition rate for sudden switching [which in (1+1) dimensions is finite]. Unlike our later results, this rate can be evaluated without requiring high-performance computing. Consider a detector suddenly switched on at time $t = 0$ and switched off at time $t = T$. From (8) (substituting Gaussian by sudden switching) the transition rate is

$$\dot{\mathcal{P}} = \frac{-\lambda^2}{2\pi} \mathrm{Re}\left(\int_0^T \mathrm{d}s\, e^{\mathrm{i}\Omega s} \log\left[1 - e^{\frac{2\pi\mathrm{i}}{aL}\left(e^{aT} - e^{a(T-s)}\right)}\right] \right) \tag{9}$$

Plotting this expression as a function of acceleration in Fig. 1 we see that the rate at which this detector clicks can decrease with growing (small) acceleration.

We find that this phenomenon persists for Gaussian switching, not only in the transition rate, but also in the transition probability itself. However the latter is trickier to evaluate numerically