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Approximate degeneracy of heavy-light mesons with the same L

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ABSTRACT

Careful observation of the experimental spectra of heavy-light mesons tells us that heavy-light mesons with the same angular momentum L are almost degenerate. The estimate is given how much this degeneracy is broken in our relativistic potential model, and it is analytically shown that expectation values of a commutator between the lowest order Hamiltonian and $\vec{L}^2$ are of the order of $1/m_Q$ with a heavy quark mass m_Q. It turns out that nonrelativistic approximation of heavy quark system has a rotational symmetry and hence degeneracy among states with the same L. This feature can be tested by measuring higher orbitally and radially excited heavy-light meson spectra for $D/D_s/B/B_s$ in LHCb and forthcoming BelleII.

1. Introduction

Ever since the discovery of $X(3872)$, $D_{s0}(2317)$, and $D_{s1}(2460)$ in 2003, there have been many more XYZ as well as higher radially and orbitally excited particles found at Belle, BESII, BESIII, BaBar, and LHCb [1]. There are a couple of problems for these particles. One is that most of them appear at thresholds and hence there may be kinematical explanations possible. Another point is that some of them should be multiquark states because they cannot be explained as higher excited states of ordinary quarkonium due to the charged states.

When focusing on higher orbital excitations of the heavy-light system, we see some tendency of their spectroscopy which has not yet been explained by heavy quark symmetry. The problem is described as follows. Even though the angular momentum L is not a good quantum number in the heavy quark system, it seems that masses of states with the same L are close to each other even for the heavy-light system.

To explain this approximate degeneracy among heavy-light mesons with the same L observed in experiments, we need to show, at least analytically or numerically, how small matrix elements of this resultant difference operator are. One of the powerful quark models is the relativized Godfrey–Isgur (GI) model [2,3] in which their lowest order Hamiltonian commutes with $\vec{L}$ even in their relativized formulation. Hence, there is no wonder within their formulation why the masses with the same L are close to each other. However, when calculating commutator of the lowest order Hamiltonian and $\vec{L}$ in our relativistic potential model [4,5], we obtain nonvanishing result. Difference between the GI and our models is in that we cast a light quark into a four-component Dirac spinor which causes non-vanishing commutator as seen below while the GI treats it a two-component spinor.

In the past decades, the heavy-light meson families have become a rich structure as seen in PDG [1]. Even though it does not take into account the heavy quark symmetry, the GI model [2,3] has been successful in reproducing and predicting low lying hadrons and heavy-light mesons except for D_{sJ}. This model respects angular momentum conservation at the lowest order so that states with the same angular momentum L are degenerate without spin-orbit interactions.

Let us look at numerical results of models only for D mesons which include a heavy quark c and compare them with each other and with experimental data in Table 1. A model in the second column [2,6,7] is the GI model itself and a model in the seventh column [8,9] is a nonrelativistic potential model including a one-loop computation of the heavy-quark interaction. Those in the third column [10,11] use the Bethe–Salpeter formulation to expand the system in terms of $1/m_Q$, while ours in the sixth column [4] uses the Foldy–Wouthouysen–Tani transformation to obtain the equation of

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Table 1
The D meson masses in MeV from different quark models and experimental data. Models of ZVR [10], DE [11], EFG [12], and MMS [4] respect heavy-quark symmetry.

State	GI [2,6,7]	ZVR [10]	DE [11]	EFG [12]	MMS [4]	LS [8,9]	EXP [13–17]	Average	Gap
$D(1^1S_0)$	1874	1850	1868	1871	1869	1867	1867	1938	
$D(1^3S_1)$	2038	2020	2005	2010	2011	2010	2009		
$D(1^3P_0)$	2398	2270	2377	2406	2283	2252	2361	2394	456
$D_1(1P)$	2455	2400	2417	2426	2421	2402	2427		
$D'_1(1P)$	2467	2410	2490	2469	2425	2417	2422	2443	49
$D(1^3P_2)$	2501	2460	2460	2460	2468	2466	2463		
$D(1^3D_1)$	2816	2710	2795	2788	2762	2740	2781	2763	320
$D_2(1D)$	2816	2740	2775	2806	2800	2693	2745		
$D'_2(1D)$	2845	2760	2833	2850	–	2789	2745	2763	0
$D(1^3D_3)$	2833	2780	2799	2863	–	2719	2800/2762		
$D(1^3F_2)$	3132	3000	3101	3090	–	–	–		
$D_3(1F)$	3109	3010	3074	3129	–	–	–		
$D'_3(1F)$	3144	3030	3123	3145	–	–	–		
$D(1^3F_4)$	3113	3030	3091	3187	–	–	–		
$D(1^3G_3)$	3398	3240	–	3352	–	–	–		
$D_4(1G)$	3365	3240	–	3403	–	–	–		
$D'_4(1G)$	3400	3260	–	3415	–	–	–		
$D(1^3G_5)$	3362	–	–	3473	–	–	–		
$D(2^1S_0)$	2583	2500	2589	2581	–	2555	2560	2595	
$D(2^3S_1)$	2645	2620	2692	2632	–	2636	2692		
$D(2^3P_0)$	2932	2780	2949	2919	–	2752	–		
$D_1(2P)$	2925	2890	2995	2932	–	2886	–		
$D'_1(2P)$	2961	2890	3045	3021	–	2926	–		
$D(2^3P_2)$	2957	2940	3035	3012	–	2971	–		
$D(2^3D_1)$	3232	3130	–	3228	–	3168	–		
$D_2(2D)$	3212	3160	–	3259	–	3145	–		
$D'_2(2D)$	3249	3170	–	3307	–	3215	–		
$D(2^3D_3)$	3227	3190	–	3335	–	3170	–		
$D(2^3F_2)$	3491	3380	–	–	–	–	–		
$D_3(2F)$	3462	3390	–	–	–	–	–		
$D'_3(2F)$	3499	3410	–	–	–	–	–		
$D(2^3F_4)$	3466	3410	–	3610	–	–	–		
$D(2^3G_3)$	3722	–	–	–	–	–	–		
$D_4(2G)$	3687	–	–	–	–	–	–		
$D'_4(2G)$	3723	–	–	–	–	–	–		
$D(2^3G_5)$	3685	–	–	3860	–	–	–		

motion for a $Q\bar{q}$ bound system and is essentially the same formulation as that of Ref. [10]. Hence the following arguments given in Sect. 2 can be derived from Refs. [10,11], too. Finally Ref. [12] uses a quasipotential approach whose details are given in their paper. Similar tables for $D_s/B/B_s$ mesons can be easily obtained and they give tendency similar to Table 1. Because we would like to extract and show the essence of our claim, we omit them in this article. It is not amazing to see that states with the same L of the GI model have similar mass values for states with the same L because it respects L. However, it is surprising that even models respecting heavy quark symmetry produce the results similar to the GI model, which can be seen from Table 1.

States in Table 1 are assigned definite values of $^{2S+1}L_J$ in the first column. Even though our relativistic wave function is not an eigenstate of L in our formulation [4], we can still assign $^{2S+1}L_J$ to each state in the nonrelativistic limit.

In the last two columns of Table 1, we give average values of experimental data within a spin doublet of the heavy-quark system and gap values between spin doublets. For instance, average values are given by 1938 MeV for a spin multiplet ($J^P = 0^-, 1^-$), 2394 MeV for a multiplet $(0^+, 1^+)$, 2443 MeV for a multiplet $(1^+, 2^+)$, 2763 MeV for a multiplet $(1^-, 2^-)$, etc. Gap values are given by difference of these values, i.e., 456 MeV between multiples $(0^-, 1^-)$ with $L = 0$ and $(0^+, 1^+)$ with $L = 1$, 49 MeV between $(0^+, 1^+)$ and $(1^+, 2^+)$ with the same $L = 1$, 320 MeV between $(1^+, 2^+)$ with $L = 1$ and $(1^-, 2^-)$ with $L = 2$, etc. We can see that mass differences within a spin doublet and between doublets with the same L are very small compared with a mass gap between different multiplets with different L, which is nearly equal to the value of the QCD $\Lambda_{QCD} \sim 300$ MeV[1] [1] for $n_f = 4$.

2. Analytical analysis

Using the heavy quark symmetry, the lowest order Hamiltonian in our relativistic potential model [4,5] is given by

$$H_0 = \vec{\alpha}_q \cdot \vec{p} + m_q \beta_q, \tag{1}$$

whose commutation relation with $\vec{L} = \vec{r} \times \vec{p}$ is given by

$$[H_0, L_i] = -i\left(\vec{\alpha}_q \times \vec{p}\right)_i. \tag{2}$$

On the other hand, we have the following commutation relation,

$$\left[H_0, \frac{1}{2}\Sigma_{qi}\right] = i\left(\vec{\alpha}_q \times \vec{p}\right)_i, \tag{3}$$

with a light quark spin $\vec{\Sigma}_q/2$. Adding Eqs. (2) and (3), we obtain conservation of $\vec{j}_\ell = \vec{L} + \vec{\Sigma}_q/2$ of light-quark degrees of freedom as

[1] We expect that a gap value is roughly $\Lambda_{QCD} \sim 300$ MeV because this gap is caused by strong interaction characterized by Λ_{QCD}, which is numerically shown in Ref. [18] when deriving mass gap relation between two spin multiplets. In Ref. [1], the notation $\Lambda_{\overline{MS}}$ is taken instead of Λ_{QCD}.