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# Radiatively induced breaking of conformal symmetry in a superpotential

A.B. Arbuzov [a,b], D.J. Cirilo-Lombardo [a,c,*]

[a] Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia
[b] Department of Higher Mathematics, Dubna State University, 141982 Dubna, Russia
[c] National Institute of Plasma Physics (INFIP-CONICET), Department of Physics, FCEyN, Universidad de Buenos Aires, Buenos Aires 1428, Argentina

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ABSTRACT

Radiatively induced symmetry breaking is considered for a toy model with one scalar and one fermion field unified in a superfield. It is shown that the classical quartic self-interaction of the superfield possesses a quantum infrared singularity. Application of the Coleman–Weinberg mechanism for effective potential leads to the appearance of condensates and masses for both scalar and fermion components. That induces a spontaneous breaking of the initial classical symmetries: the supersymmetry and the conformal one. The energy scales for the scalar and fermion condensates appear to be of the same order, while the renormalization scale is many orders of magnitude higher. A possibility to relate the considered toy model to conformal symmetry breaking in the Standard Model is discussed.



## 1. Introduction

It was shown [1] that infrared divergences in quantum loop contributions to effective potentials of various models in quantum field theory (QFT) can lead to the necessity to introduce a non-zero renormalization scale and thus generate a spontaneous breaking of the conformal symmetry. In particular, the $\phi^4$ model as well as Abelian and Yang–Mills gauge models were considered in [1]. Here we will apply the Coleman–Weinberg (CW) mechanism to a simple QFT model for a superfield which joins scalar and fermion physical fields, see for example [2,3] for application of the CW mechanism in different physical scenarios.

From the phenomenological point of view our study is motivated by the recent discovery of the Higgs boson. The observed properties of the latter are in a good agreement with the Standard Model (SM) predictions. Nevertheless the origin of the electroweak energy scale is still unclear. For the time being it is just introduced from the beginning into the Lagrangian of the SM as the tachyon-like mass parameter. On the other hand the electroweak (EW) energy scale of about 100 GeV is seen both in the Higgs (and electroweak sector) and the top quark mass. The relation $4m_H^2 \approx 2m_t^2 \approx v^2$ between the observed Higgs boson mass $m_H$, the top quark mass $m_t$, and the Higgs boson vacuum expectation value $v$ holds with a high accuracy [4]. In any case, the coincidence of the scales is an intriguing puzzle. Another face of the electroweak scale puzzle is the hierarchy problem of the SM due to quadratic divergences in the running of the Higgs boson mass within the SM. In fact, renormalization of $m_H$ suffers from *fine tuning* between the (loop) contributions due to the top quark, the Higgs boson, and EW bosons (note that only longitudinal components of $W$ and $Z$ bosons, *i.e.* the scalar Goldstones, contribute). It is well known that resolution of the fine tuning problem can be done by a supersymmetric extension of the SM. In any case, a certain (symmetry?) relation between fermionic and bosonic contribution is required to solve the problem.

On the other hand there are many indirect indications of the Conformal Symmetry (CS) might be the proper symmetry of the underlying fundamental theory, while the SM is just an effective model emerged after a spontaneous breaking of the CS, see e.g. Refs. [5,6].

In Ref. [7], the possibility to generate a soft breaking of the conformal symmetry in the sector of the SM which joins the Higgs boson and the top quark was discussed. In fact, the infrared singularity is present in this system and the Coleman–Weinberg mechanism can be applied. Nevertheless, the question about the relation between renormalization conditions for the scalar and the fermion field remains unsolved. As discussed, a certain *bootstrap* should happen in the SM between the Higgs boson and the top quark. In this paper we suggest to look what comes out if the two fields are joined into a superfield. Of course, it is just one of many other possibilities but it provides a certain feeling of the bootstrap.

* Corresponding author.
*E-mail address:* diego777jcl@gmail.com (D.J. Cirilo-Lombardo).

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It is frequent to find in the current research proposals that suggest (supersymmetric) extensions of the Standard Model with introduction of a hidden sector that shows conformal invariance above a certain high energy scale (see in a different context Ref. [8]), and it couples to the SM sector by some higher dimensional operators. The nontrivial conformally invariant hidden sector leads to a novel type of observable effects in the SM sector, which may be accessible in near future experiments at a TeV scale. On the other hand, since one of the most appealing new physics at the TeV scale is the supersymmetry (SUSY), it is very natural to consider supersymmetric extension of the Standard Model considering the introduction of superfields containing the particles involved in the interaction of interest. The first aim of this paper is to investigate the supersymmetric extension of the model based on the superconformal field theory by means of introduction of a scalar superfield, as we will show in Section 2. It is well known that the four dimensional superconformal field theory is powerful enough to obtain the crucial dynamical information about the physics of particle interaction due to the fact that the interaction itself is hidden inside quadratic and dynamical terms in the Lagrangian of the theory. For example, the relation between the R-charge and the conformal dimension determines the conformal dimensions of the chiral operators beyond the perturbation theory. We also have more severe inequalities for conformal dimensions that are not available in non-supersymmetric theories. In this sense, the introduction of the SUSY is theoretically well motivated. Physically the interplay between conformal symmetry and supersymmetry and their breaking (of both or of any of them, total or partial) introduces automatically extra constraints on particle physics.

As we pointed out before, previous investigations on the particle physics within the context of (super) conformal models assume that a certain particle sector remains conformal at least down to the electroweak scale, at which any experimental evidence is expected. The problem is claimed usually of a partial breaking of SUSY or conformal symmetry and how to conciliate both. It is usually solved by means of a gauge mediation or by tuning the Kähler potential. However there are no such problems through our paper, consequently this particular point not will be analyzed here.

## 2. Conformal models and supersymmetry

The hints in order to introduce the fermionic interactions in any classical bosonic action endowed by conformal symmetry were presented for the first time in the seminal papers of Akulov and Pashnev [9] where the starting point was the well know the AFF (de Alfaro, Fubini and Furlan) conformal model [10]. Without going into details (see [9]), the idea was to introduce a superfield having the form

$$\Phi = \varphi + i\theta^{\alpha}\psi_{\alpha} + i\theta^{\alpha}\theta_{\alpha}F\,, \quad (\theta^{\alpha}\theta_{\alpha} = \theta\overline{\theta} \text{ etc.}) \tag{1}$$

into the following general n-dimensional action with the standard super-kinetic term

$$S_{kin} = -\frac{1}{32}\int d^n x d^2\theta\; D^2\overline{D}^2\left(\overline{\Phi}\Phi\right) \tag{2}$$

where $\Phi$ is the chiral superfield (with standard anti-chiral counterpart $\overline{\Phi}$) and the super-derivatives are usually defined as $D_{\alpha}\Phi \equiv \left(\frac{\partial}{\partial\theta^{\alpha}} + i\left(b_{\alpha\beta}\theta^{\beta}\right)^{i}\frac{\partial}{\partial x^{i}}\right)\Phi$ (similarly for $\overline{D}_{\dot{\alpha}}\overline{\Phi}$) where $b_{\alpha\beta}$ is a symmetric matrix fixed by the symmetry properties of the superspace under consideration, e.g. by supercharges. The usual conventions for down and up indices of the fermionic variables with $\epsilon^{12} = \epsilon_{12} = 1$, $(\alpha, \beta = 1, 2)$ are assumed (for the dotted indices $\dot{\alpha}, \dot{\beta} = \dot{1}, \dot{2}$ are similarly related, as usual), also for spacetime indices: $i, j = 0, ...., d-1$. The component form of expression (2) is obtained by inserting (1) into (2) and integrating over the Grassman variables:

$$S_{kin} = -\frac{1}{2}\int d^n x\left(\partial_i\varphi\partial^i\overline{\varphi} - i\left(\overline{\psi}^{\dot{\alpha}} b_{\dot{\alpha}\beta}\right)_j \partial^j\psi^{\beta} + 4F\overline{F}\right). \tag{3}$$

The interaction part was defined in the general form as

$$S_{int} = \int d^n x d^2\theta\, d^2\overline{\theta}\, V(\Phi\overline{\Phi}). \tag{4}$$

Without loss of generality the simplest 4-dimensional case will be treated. Remind now the effective potential for a scalar field with a $\varphi^4$ interaction, which was derived by S. Coleman and E. Weinberg [1] in the one-loop approximation

$$U(\varphi) = \frac{\lambda}{4!}\varphi^4 + \frac{\lambda^2}{256\pi^2}\varphi^4\left[\ln\left(\frac{\varphi^2}{M^2}\right) - \frac{25}{6}\right]. \tag{5}$$

The presence of the renormalization scale $M$ indicates the radiatively induced breaking of the conformal symmetry in this model.

We can pass from the bosonic effective potential to the supersymmetric one by introducing the superfield. Note that due that the standard version of the four-dimensional supersymmetry, the simplest superfields contain a complex Lorentz scalar and a chiral (left-handed or right-handed) fermion. To avoid confusion henceforth we define $\langle\varphi\rangle^2 \equiv \overline{\varphi}\varphi$. Then we obtain the following expression

$$\begin{aligned} W\left(\langle\varphi\rangle, \langle\bar{\psi}\psi\rangle\right) = &\left(\langle\varphi\rangle^4 + 2\langle\varphi\rangle\langle\bar{\psi}\psi\rangle\right) \\ &\times\left[\frac{\lambda}{4!} + \frac{\lambda^2}{256\pi^2}\left(\ln\left(\frac{\langle\varphi\rangle^2}{M^2}\right) - \frac{25}{6}\right)\right] \\ &+ \langle\varphi\rangle\langle\bar{\psi}\psi\rangle\left[\frac{\lambda}{2} + \frac{\lambda^2}{256\pi^2}\right], \end{aligned} \tag{6}$$

where the Grassman integration was performed under the (physical) measure

$$\int \mu\left(\theta^2\right) d^2\theta = b \quad \text{and} \quad \int \mu\left(\theta^2\right)\theta^2 d^2\theta = a$$

$$\text{with}\;\; \mu\left(\theta^2\right) \equiv a\exp\left(b\frac{\theta^2}{a}\right),$$

where $a$ and $b$ are constants related to the group manifold structure (volume), that must be included into the above measure in order to recover the original Coleman–Weinberg potential when all fermions vanish.

Let us look for a minimum of the potential. The conditions

$$\begin{cases} \frac{\partial W}{\partial\langle\bar{\psi}\psi\rangle} = 0, \\ \frac{\partial W}{\partial\langle\varphi\rangle} = 0 \end{cases} \tag{7}$$

lead to the following solution for the scalar and fermion condensate values:

$$v^2 \equiv \langle\varphi\rangle^2 = M^2\exp\left\{-\frac{196\pi^2}{\lambda}\right\},$$

$$\langle\bar{\psi}\psi\rangle = -v^3\frac{2\lambda}{7}. \tag{8}$$

We assumed that the coupling constant $\lambda \lesssim 1$ so that the perturbative solution is reliable.