Physics Letters B 756 (2016) 6–9

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Model of the N-quark potential in $SU(N)$ gauge theory using gauge-string duality

Oleg Andreev [a,b,*]

[a] L.D. Landau Institute for Theoretical Physics, Kosygina 2, 119334 Moscow, Russia
[b] Arnold Sommerfeld Center for Theoretical Physics, LMU-München, Theresienstrasse 37, 80333 München, Germany

A R T I C L E I N F O

Article history:
Received 9 December 2015
Received in revised form 29 February 2016
Accepted 29 February 2016
Available online 3 March 2016
Editor: A. Ringwald

A B S T R A C T

We use gauge-string duality to model the N-quark potential in pure Yang–Mills theories. For $SU(3)$, the result agrees remarkably well with lattice simulations. The model smoothly interpolates between almost the Δ-law at short distances and the Y-law at long distances.

1. Introduction

Predicting properties of hadrons still represents a serious challenge for Quantum Chromodynamics (QCD). Heavy quarks closely resemble static test charges and therefore are useful to probe confining properties of QCD. So far, great progress has been made in the study of quarkonia, i.e. mesonic states that contain two heavy constituent quarks. In contrast, systems of three or more heavy quarks, which are a good starting point for understanding the phenomenology of baryons and multi-quark bound states, are much less studied. In this case a key issue is whether multi-quark interactions can be understood in terms of two-body interactions or whether there are genuine three- and many-body effects to be considered as part of the overall picture of strong interactions [1,2].

The best known phenomenological models of the N-quark potential are those of $N = 3$, the so-called Δ and Y-laws [3]. The Δ-law is based on pairwise interactions between quarks, while the Y-law is an example of three-body interactions. In the infrared region the former predicts that the potential grows linearly with the perimeter of the triangle formed by quarks [4], while the latter predicts a linear growth with the minimal length of a string network which has a junction at the Fermat point of the triangle [5].

Until recently, lattice gauge theory was the premier method for obtaining quantitative and qualitative information about strongly interacting gauge theories. For the three-quark potential the accuracy of numerical simulations has been improved during the past decade [6–9] that provided evidence for the Y-law at long distances. On the other hand, it is expected that at short distances the Δ-law is a good approximation to the potential [3,6]. However, what is still missing is a model which would incorporate the Δ-law at short distances and the Y-law at long ones.

In this Letter we present the first example of such a model. It continues a series of studies [10–12] devoted to the static potentials in four-dimensional (pure) gauge theory by means of a five (ten)-dimensional effective string theory. Our reasons for continuing to pursue this model are:

(1) Because there is no string theory which is dual to QCD. It would seem very good to gain what experience we can by solving any problems that can be solved within the effective string model already at our disposal.

(2) Because the results provided by this model are consistent with the lattice calculations and QCD phenomenology [13–15].

(3) Because analytic formulas are obtained by solving this model.

(4) Because it allows us to make predictions [16] which may then be tested by means of other methods, e.g., numerical simulations.

Before proceeding to the detailed analysis, let us set the basic framework. As for the quark–antiquark potential, the static N-quark potential can be determined from the expectation value of a Wilson loop. The loop in question, baryonic loop, is defined in a gauge-invariant manner as $W_{\mathrm{NQ}} = \frac{1}{N!}\varepsilon_{a_1\dots a_N}\varepsilon_{a'_1\dots a'_N}\prod_{i=1}^{N} U^{a_i a'_i}$, with the path-ordered exponents $U^{a_i a'_i}$ along the lines shown in Fig. 1. In the limit $T \to \infty$ the expectation value of the loop is simply $\langle W_{\mathrm{NQ}}(C)\rangle \sim \mathrm{e}^{-ET}$, with E the ground state energy of N quarks (N-quark potential).

In discussing baryonic Wilson loops, we adapt the formalism [17,18] proposed within the AdS/CFT correspondence [19] to our

* Correspondence to: L.D. Landau Institute for Theoretical Physics, Kosygina 2, 119334 Moscow, Russia.
E-mail address: andre@itp.ac.ru.

http://dx.doi.org/10.1016/j.physletb.2016.02.070
0370-2693/

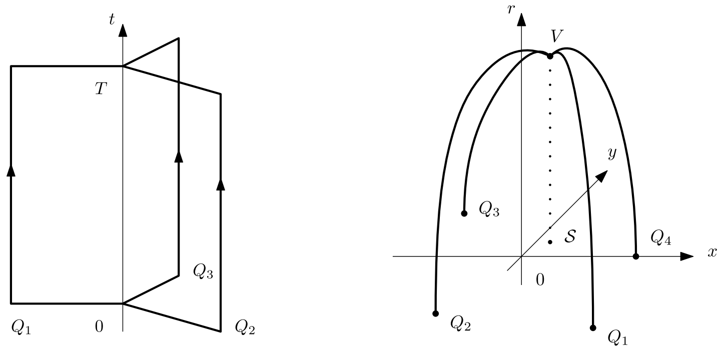

Fig. 1. Left: A baryonic Wilson loop in $SU(3)$ gauge theory. Right: In $SU(4)$, a configuration used to calculate the expectation value of a baryonic loop. The quarks are set on the x-y plane. V is a baryon vertex located at $r=r_0$ and $\mathcal{S}$ is its projection onto the x-y plane.

purposes. First, we take the following ansatz for the background geometry [20]

$$ds^2 = e^{\mathfrak{s}r^2}\frac{R^2}{r^2}\left(dt^2 + d\vec{x}^2 + dr^2\right) + e^{-\mathfrak{s}r^2} g^{(5)}_{ab}\, d\omega^a d\omega^b\,, \tag{1}$$

where $d\vec{x}^2 = dx^2 + dy^2 + dz^2$. This is a deformed product of AdS_5 and an internal space (five-sphere) $\mathbf{X}$ whose coordinates are ω^a. The deformation is due to the r-dependent warp factor, with $\mathfrak{s}$ the deformation parameter. Such a deformation is a kind of the soft wall model of [21], where the violation of conformal symmetry is manifest in the background metric. In (1), there are two free parameters to be fitted to the results of numerical simulations or quarkonia spectra. Both fits look very good [13,14].

Next, we consider the baryon vertex which is a N-string junction. Since we are interested in a static quark potential, we choose static gauge and then make an ansatz for the action, describing a static configuration, of the form

$$S_{\text{vert}} = m\frac{e^{-2\mathfrak{s}r^2}}{r}T\,, \tag{2}$$

where m and $\mathfrak{s}$ are parameters, r is independent of t, and $T = \int_0^T dt$. In what follows, we will assume that quarks are placed at points on the boundary of 5-dimensional deformed AdS (at $r=0$) but at the same point in the internal space. This assumption makes the problem effectively five-dimensional. Therefore the detailed structure of $\mathbf{X}$ is not important, except for the warp factor depending on the radial direction. The motivation for such a factor in (2) is drawn from the AdS/CFT construction, where the baryon vertex is a 5-brane [17]. Taking a term $\int dt d^5\omega\sqrt{g^{(6)}}$ from the world-volume action of the brane results in $Te^{-2\mathfrak{s}r^2}/r$ if r is independent of t. This is, of course, a heuristic argument but, as we will see, the ansatz (2) is quite successful: it allows us to describe the results for $N=3$ using just one parameter.

The expectation value of the Wilson loop is schematically given by the path integral over world-sheet fields

$$\langle W_{\text{NQ}}(C)\rangle = \int D\Psi e^{-S_w}\,, \tag{3}$$

where S_w is a total action of the Nambu–Goto strings and vertex. The strings are stretched between the quarks on the boundary and the baryon vertex in the interior, as sketched in Fig. 1. In principle, the integral can be evaluated approximately in terms of minimal surfaces that obey the boundary conditions. The result is written as $\langle W_{\text{NQ}}(C)\rangle = \sum_n w_n \exp[-S_n]$, where S_n means a renormalized minimal area whose weight is w_n.

2. Calculating the N-quark potential

We consider a situation in which N quarks are placed at the vertices of a regular N-sided convex polygon of side length L. This configuration has the symmetry group D_N. Hence $\mathcal{S}$ is a center of the polygon and all the strings have an identical profile. To compute the potential, we proceed along very similar lines to those of [12]. First, we take the static gauge that allows us to solve the equations of motion and determine the string profile. Next we extremize the action with respect to the location of the baryon vertex r_0 that results in the no-force condition at $r=r_0$. There is, however, one important distinction between the present calculation and those in the literature devoted to large N gauge theories. We make an assumption that the parameter m is negative. As a result, gravity pulls the vertex toward the boundary. This bends the strings and blunts the tip of the configuration [16], as shown in Fig. 1.

Having found the solution, we can compute the total energy of the configuration. At the end of the day we arrive at [16]

$$L(\nu) = 2\sin\left(\frac{\pi}{N}\right)\sqrt{\frac{\lambda}{\mathfrak{s}}}\Biggl[\int_0^1 dv\, v^2 e^{\lambda(1-v^2)}\left(1 - v^4 e^{2\lambda(1-v^2)}\right)^{-\frac{1}{2}} + \int_{\sqrt{\frac{\nu}{\lambda}}}^1 dv\, v^2 e^{\lambda(1-v^2)}\left(1 - v^4 e^{2\lambda(1-v^2)}\right)^{-\frac{1}{2}}\Biggr] \tag{4}$$

and

$$E(\nu) = N\mathfrak{g}\sqrt{\frac{\mathfrak{s}}{\lambda}}\Biggl[\kappa\sqrt{\frac{\lambda}{\nu}}e^{-2\nu} - 1 + \int_0^1 \frac{dv}{v^2}\left(e^{\lambda v^2}\left(1 - v^4 e^{2\lambda(1-v^2)}\right)^{-\frac{1}{2}} - 1\right) + \int_{\sqrt{\frac{\nu}{\lambda}}}^1 \frac{dv}{v^2}\, e^{\lambda v^2}\left(1 - v^4 e^{2\lambda(1-v^2)}\right)^{-\frac{1}{2}}\Biggr] + C\,, \tag{5}$$

where $\nu = \mathfrak{s}r_0^2$, $\mathfrak{g} = \frac{R^2}{2\pi\alpha'}$, $\kappa = \frac{m}{N\mathfrak{g}}$, and C is a normalization constant. λ is a function of ν and κ such that $\lambda = -\text{ProductLog}[-\nu e^{-\nu}(1-\kappa^2(1+4\nu)^2 e^{-6\nu})^{-\frac{1}{2}}]$, where ProductLog$(z)$ is the principal solution for w in $z = we^w$.[1] Also note that $\nu \in [0, \nu_*]$, with ν_* a solution to $\nu^2 = e^{2(\nu-1)}(1-\kappa^2(1+4\nu)^2 e^{-6\nu})$.

[1] See, e.g., https://reference.wolfram.com/language/ref/Pro-ductLog.html.