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The Hawking cascades of gravitons from higher-dimensional Schwarzschild black holes

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ABSTRACT

It has recently been shown that the Hawking evaporation process of $(3+1)$-dimensional Schwarzschild black holes is characterized by the dimensionless ratio $\eta \equiv \tau_{\text{gap}}/\tau_{\text{emission}} \gg 1$, where τ_{gap} is the characteristic time gap between the emissions of successive Hawking quanta and τ_{emission} is the characteristic timescale required for an individual Hawking quantum to be emitted from the Schwarzschild black hole. This strong inequality implies that the Hawking cascade of gravitons from a $(3+1)$-dimensional Schwarzschild black hole is extremely *sparse*. In the present paper we explore the semi-classical Hawking evaporation rates of *higher*-dimensional Schwarzschild black holes. We find that the dimensionless ratio $\eta(D) \equiv \tau_{\text{gap}}/\tau_{\text{emission}}$, which characterizes the Hawking emission of gravitons from the $(D+1)$-dimensional Schwarzschild black holes, is a *decreasing* function of the spacetime dimension. In particular, we show that higher-dimensional Schwarzschild black holes with $D \gtrsim 10$ are characterized by the relation $\eta(D) < 1$. Our results thus imply that, contrary to the $(3+1)$-dimensional case, the characteristic Hawking cascades of gravitons from these higher-dimensional black holes have a *continuous* character.

1. Introduction

Analyzing the dynamics of quantum fields in curved black-hole spacetimes, Hawking [1] has reached the intriguing conclusion that black holes have a well defined temperature [2,3]. In particular, Hawking [1] has revealed that semi-classical black holes are characterized by quantum emission spectra which have distinct thermal features [4]. This remarkable prediction is certainly one of the most important outcomes of the interplay between quantum field theory and classical general relativity.

It has recently been shown [5] that the semi-classical Hawking radiation flux out of a $(3+1)$-dimensional Schwarzschild black hole is extremely sparse [see also [6–8] for earlier discussions of this characteristic property of the $(3+1)$-dimensional Schwarzschild black hole]. In particular, the Hawking emission of gravitons out of a $(3+1)$-dimensional Schwarzschild black hole is characterized by the dimensionless large ratio [5]

$$\eta \equiv \frac{\tau_{\text{gap}}}{\tau_{\text{emission}}} \gg 1 \,, \tag{1}$$

where τ_{gap} is the characteristic time gap between the emissions of successive black-hole gravitational quanta [see Eqs. (10) and (11) below], and τ_{emission} is the characteristic timescale required for an individual Hawking quantum to be emitted from the black hole [see Eq. (12) below].

The dimensionless large ratio (1) implies that the semi-classical Hawking evaporation of $(3+1)$-dimensional Schwarzschild black holes is indeed sparse. That is, the characteristic time gap between the emissions of successive gravitational quanta out of an evaporating $(3+1)$-dimensional Schwarzschild black hole is very large on the natural timescale $2\pi/\omega$ [see Eq. (12) below] set by the characteristic energy (frequency) of the emitted Hawking quanta. The characteristic large ratio (1) therefore suggests a simple physical picture in which an evaporating $(3+1)$-dimensional Schwarzschild black hole [9,10] typically emits Hawking quanta one at a time [5–8].

One naturally wonders whether the strong inequality (1), which characterizes the Hawking evaporation process of the $(3+1)$-dimensional Schwarzschild black hole, is a generic feature of all $(D+1)$-dimensional Schwarzschild black-hole spacetimes? In order to answer this interesting question, we shall explore in this paper the semi-classical Hawking evaporation of higher-dimensional Schwarzschild black holes. Below we shall show that the dimen-

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sionless ratio $\eta(D) \equiv \tau_{\text{gap}}/\tau_{\text{emission}}$, which characterizes the Hawking evaporation process of $(D+1)$-dimensional Schwarzschild black holes, is a *decreasing* function of the spacetime dimension. In particular, we shall show that higher-dimensional Schwarzschild black holes with $D \gtrsim 10$ are characterized by the relation $\eta(D) < 1$. Thus, our analysis (to be presented below) reveals the fact that the Hawking cascades of gravitons from these higher-dimensional evaporating black holes have a *continuous* character.

2. The Hawking evaporation process of $(D+1)$-dimensional Schwarzschild black holes

We study the semi-classical Hawking emission of gravitational quanta by higher-dimensional Schwarzschild black holes. The characteristic Bekenstein–Hawking temperature of an evaporating $(D+1)$-dimensional Schwarzschild black hole is given by [11]

$$T^D_{\text{BH}} = \frac{(D-2)\hbar}{4\pi r_{\text{H}}} \,, \tag{2}$$

where r_{H} is the black-hole horizon radius [12–15].

The radiation flux (that is, the number of quanta emitted per unit of time) and the radiation power (that is, the energy emitted per unit of time) for one bosonic degree of freedom out of a $(D+1)$-dimensional Schwarzschild black hole are given respectively by the integral relations [1,16,17]

$$\mathcal{F}^D_{\text{BH}} = \frac{1}{(2\pi)^D}\sum_j\int_0^{\infty} dV_D(\omega)\frac{\Gamma}{e^{\hbar\omega/T_{\text{BH}}}-1} \tag{3}$$

and

$$\mathcal{P}^D_{\text{BH}} = \frac{1}{(2\pi)^D}\sum_j\int_0^{\infty} dV_D(\omega)\frac{\Gamma\hbar\omega}{e^{\hbar\omega/T_{\text{BH}}}-1} \,, \tag{4}$$

where j denotes the angular harmonic indices of the emitted field mode. The frequency-dependent coefficients $\Gamma = \Gamma(\omega; j, D)$ are the dimensionless greybody factors [17] which quantify the partial scattering of the emitted field modes by the effective curvature potential that surrounds the $(D+1)$-dimensional Schwarzschild black hole. These factors are determined by the higher-dimensional version of the Regge–Wheeler equation [18]

$$\Big(\frac{d^2}{dr_*^2} + \omega^2 - V\Big)\phi = 0 \,, \tag{5}$$

where r_* is a 'tortoise' radial coordinate which is determined by the relation $dr_*/dr = [1-(r_{\text{H}}/r)^{D-3}]^{-1}$. The effective curvature potential in the Schrödinger-like wave equation (5) is given by [18]

$$V(r;D) = \Big[1-\Big(\frac{r_{\text{H}}}{r}\Big)^{D-3}\Big]\Big[\frac{l(l+D-2)+(D-1)(D-3)/4}{r^2} + \frac{(1-p^2)(D-1)^2 r_{\text{H}}^{D-2}}{4r^D}\Big] \,, \tag{6}$$

where l is the angular harmonic index of the perturbation mode and $p = 0, 2$ for gravitational tensor perturbations and gravitational vector perturbations, respectively [18,19].

Substituting into (3) and (4) the expression

$$dV_D(\omega) = [2\pi^{D/2}/\Gamma(D/2)]\omega^{D-1}d\omega \tag{7}$$

for the D-dimensional volume in frequency-space of the shell $(\omega, \omega + d\omega)$, one finds

$$\mathcal{F}^D_{\text{BH}} = \frac{1}{2^{D-1}\pi^{D/2}\Gamma(D/2)}\sum_j\int_0^{\infty}\Gamma\frac{\omega^{D-1}}{e^{\hbar\omega/T_{\text{BH}}}-1}d\omega \tag{8}$$

and

$$\mathcal{P}^D_{\text{BH}} = \frac{\hbar}{2^{D-1}\pi^{D/2}\Gamma(D/2)}\sum_j\int_0^{\infty}\Gamma\frac{\omega^{D}}{e^{\hbar\omega/T_{\text{BH}}}-1}d\omega \tag{9}$$

for the semi-classical Hawking radiation flux and the semi-classical Hawking radiation power which characterize the evaporating $(D+1)$-dimensional Schwarzschild black holes.

3. The characteristic timescales of the Hawking evaporation process

An important timescale which characterizes the Hawking evaporation process of the $(D+1)$-dimensional Schwarzschild black holes is given by the time gap between the emissions of successive Hawking quanta. There are several distinct (though closely related) ways to quantify this fundamental time scale. Here we shall use two natural definitions for this characteristic time gap:

(1) One can use the reciprocal of the black-hole radiation flux (8) in order to quantify the characteristic time gap between the emissions of successive Hawking quanta. That is,

$$\tau^{(1)}_{\text{gap}} = \frac{1}{\mathcal{F}^D_{\text{BH}}} \,. \tag{10}$$

(2) One can also use the reciprocal of the black-hole radiation power (9) and the characteristic peak frequency ω_{peak} of the semi-classical Hawking radiation spectrum in order to quantify the characteristic time gap between the emissions of successive Hawking quanta. That is,

$$\tau^{(2)}_{\text{gap}} = \frac{\omega_{\text{peak}}}{\mathcal{P}^D_{\text{BH}}} \,. \tag{11}$$

Below we shall show that the characteristic time gaps obtained from these two definitions [Eqs. (10) and (11)] are of the same order of magnitude.

A distinct timescale which characterizes the Hawking evaporation process of the $(D+1)$-dimensional Schwarzschild black holes is given by the time τ_{emission} required for an individual Hawking quantum to be emitted from the evaporating black hole. This fundamental timescale can be bounded from below by the time-period it takes to the characteristic wave field emitted from the black hole to complete a full oscillation cycle [5,10]. That is,

$$\tau_{\text{emission}} \geq \tau_{\text{oscillation}} = \frac{2\pi}{\omega_{\text{peak}}} \,. \tag{12}$$

Using the natural timescales (10), (11), and (12), one can define the fundamental dimensionless ratio

$$\eta^{(i)} \equiv \frac{\tau^{(i)}_{\text{gap}}}{\tau_{\text{emission}}} \tag{13}$$

which provides important information about the Hawking evaporation process of the semi-classical black holes [20]. In particular, physical situations which are characterized by the relation $\eta \gg 1$ describe Hawking evaporation processes which are extremely *sparse* (that is, the individual Hawking quanta emitted from the black hole are well separated in time), whereas physical situations which are characterized by the relation $\eta \ll 1$ describe Hawking evaporation processes which are effectively *continuous*.

In the next sections we shall investigate the functional dependence of the dimensionless ratios $\eta^{(i)}(D) \equiv \tau^{(i)}_{\text{gap}}/\tau_{\text{emission}}$ on the spacetime dimension $D+1$ of the evaporating black-hole spacetime.