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Large-N QCD and the Veneziano amplitude

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A R T I C L E I N F O

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A B S T R A C T

We consider four scalar mesons scattering in large-N_c QCD. Using the worldline formalism we show that the scattering amplitude can be written as a formal sum over Wilson loops. The AdS/CFT correspondence maps this sum into a sum over string worldsheets in a confining background. We then argue that for well separated mesons the sum is dominated by flat space configurations. Under additional assumptions about the dual string path integral we obtain the Veneziano amplitude.

The 1968 seminal paper by Veneziano marks the birth of string theory ("dual resonance model"). It concerns a proposal for the scattering amplitude of four scalar mesons, as follows [1]

$$\mathcal{A}(k_1,k_2,k_3,k_4) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \qquad (1)$$

with $\alpha(s)=\alpha(0)+\alpha' s$, $\alpha(t)=\alpha(0)+\alpha' t$ and s, t are Mandelstam variables.

While the amplitude (1) is appealing due to its theoretical and phenomenological properties, it lacks a derivation from Quantum Chromodynamics (QCD). It is important to clarify the exact relation between the above amplitude and QCD. In particular, we wish to find whether there exists an approximation where we can derive the amplitude (1) from QCD. The topology of the string diagram ("disk amplitude") suggests that it should be related to large-N_c QCD with fixed N_f, namely to the 't Hooft limit of QCD where quark loops are suppressed (quarks are "quenched"). As we shall see, the route from QCD to (1) involves additional approximations which are not controlled by QCD parameters. In particular, we assume that Wilson loops are calculated by flat space string worldsheets.

The first attempt to relate the Veneziano amplitude to field theory was made in 1970 [2,3], even before the birth of QCD. Sakita and Virasoro argued [3], using a scalar field theory as a model of strong interactions, that the field theory amplitude should look like a dense fishnet, see Fig. (1). Moreover, they argued that at high orders in perturbation theory the gaps between the holes in the fishnet close and the amplitude should resemble a string amplitude, namely the value of the amplitude should correspond to the area of the string worldsheet. While this idea can easily be implemented in large-N_c QCD (with a fishnet made of gluons), it cannot be the full story as it lacks the essential non-perturbative effects which are responsible for confinement with a string tension and a mass gap. It is therefore necessary to address the problem by using a formalism that incorporates non-perturbative effects.

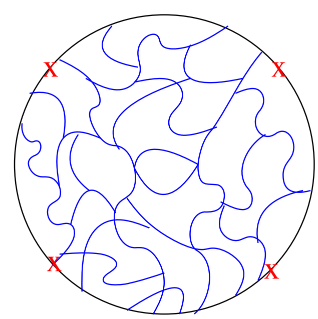

Fig. 1. Perturbative description of the four mesons scattering amplitude as a "fishnet" diagram in planar QCD.

We consider $SU(N_c)$ QCD with N_f massless quarks in the 't Hooft limit, where g^2N_c and N_f are kept fixed. The partition function takes the form

$$\mathcal{Z} = \int DA_\mu \exp(-S_{\rm YM}) \left(\det(i\not{D})\right)^{N_f} . \qquad (2)$$

Let us write the fermionic determinant as a sum over (super-)Wilson loops by using the worldline formalism [4]

$$\left(\det(i\not{D})\right)^{N_f} = \exp(N_f \Gamma[A_\mu]) , \qquad (3)$$

with

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$$\Gamma[A_\mu] = -\frac{1}{2}\int_0^\infty \frac{dT}{T}$$
$$\times \int \mathcal{D}x^\mu \mathcal{D}\psi^\mu \exp\left\{-\int_\epsilon^T d\tau \left(\frac{1}{2}\dot{x}^\mu\dot{x}^\mu + \frac{1}{2}\psi^\mu\dot{\psi}^\mu\right)\right\}$$
$$\times \mathrm{Tr}\,\mathcal{P}\exp\left\{i\int_0^T d\tau \left(A_\mu\dot{x}^\mu - \frac{1}{2}\psi^\mu F_{\mu\nu}\psi^\nu\right)\right\}, \quad (4)$$

where $\mu, \nu = 0, \dots, 3$.

The expansion of the exponent in powers of N_f

$$\exp(N_f\Gamma) \sim 1 + N_f\Gamma + \mathcal{O}(N_f^2) \quad (5)$$

is in fact an expansion in N_f/N_c, since $\langle W_1 W_2 \dots W_l\rangle_{\mathrm{conn.}} \sim N_c^2(N_f/N_c)^l$. Therefore, as we shall see, in the 't Hooft limit the dominant class of diagrams for meson scattering involves one Wilson loop[5].

Let us consider the scattering amplitude of four scalar mesons

$$\mathcal{A}(x_1, x_2, x_3, x_4) = \langle \bar{q}q(x_1)\bar{q}q(x_2)\bar{q}q(x_3)\bar{q}q(x_4)\rangle\,. \quad (6)$$

For simplicity we assume flavor-singlet mesons. Flavoured mesons can be easily incorporated by adding "Chan–Paton" factors, resulting in an overall group theoretic factor of $\mathrm{Tr}(\lambda^1\lambda^2\lambda^3\lambda^4)$. In order to calculate the amplitude we add to the action a source of the form $\int d^4x\, J(x)\bar{q}q$

$$\mathcal{Z}_J = \int DA_\mu \exp(-S_{\mathrm{YM}})\left(\det(i\not{D} + J(x))\right)^{N_f} \quad (7)$$

and differentiate the partition function with respect to $J(x_1)$, $J(x_2)$, $J(x_3)$, $J(x_4)$

$$\langle\bar{q}q(x_1)\bar{q}q(x_2)\bar{q}q(x_3)\bar{q}q(x_4)\rangle\rangle = \frac{\delta}{\delta J(x_1)}\frac{\delta}{\delta J(x_2)}\frac{\delta}{\delta J(x_3)}\frac{\delta}{\delta J(x_4)}\log\mathcal{Z}_J|_{J=0}\,. \quad (8)$$

In the worldline formalism, such a variation excludes Wilson loops whose contours do not pass through $\{x_1, x_2, x_3, x_4\}$ [6,5].

Thus in the 't Hooft limit the leading large-N_c expression for the scattering amplitude can schematically be written as

$$\mathcal{A}(x_1, x_2, x_3, x_4) = \frac{1}{2}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x \exp\left(-\int d\tau\, \tfrac{1}{2}\dot{x}_\mu^2\right)\langle W(x_1, x_2, x_3, x_4)\rangle_{\mathrm{YM}}\,. \quad (9)$$

In order to simplify the notation we omitted the worldline fermions in the above expression. The Wilson loops are calculated in the large-N pure Yang–Mills theory. The sum is over all sizes and shapes of Wilson loops that pass through the points $\{x_1, x_2, x_3, x_4\}$. The QCD amplitude admits the topology of a disk and it resembles the string disk amplitude (see Fig. 2) [7].

In order to proceed we use holography to evaluate $\langle W(x_1, x_2, x_3, x_4)\rangle_{\mathrm{YM}}$ (instead we could have used the lattice strong coupling expansion). The holographic prescription for evaluating the Wilson loop expectation value is to find the minimal worldsheet that ends on the boundary of the space (anti-de Sitter in the case of $\mathcal{N} = 4$ super Yang–Mills theory) and passes through $\{x_1, x_2, x_3, x_4\}$ [8,9].

We therefore *propose* that the field theory path integral over all sizes and shapes of contours that pass through $\{x_1, x_2, x_3, x_4\}$, translates into the Polyakov path integral for string worldsheets with a disk topology that end on the boundary of the space and pass through $\{x_1, x_2, x_3, x_4\}$

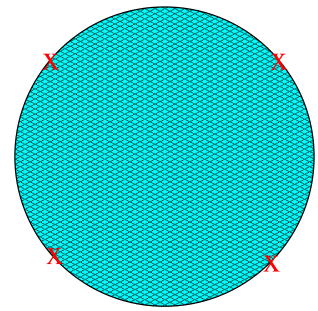

Fig. 2. Worldline description of the four mesons scattering amplitude. It resembles the string disk amplitude.

$$\frac{1}{2}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x^\mu(\tau)\exp\left(-\int d\tau\,\tfrac{1}{2}\dot{x}_\mu^2\right)\langle W(x_1, x_2, x_3, x_4)\rangle$$
$$= \int \mathcal{D}g^{\alpha\beta}\,\mathcal{D}x^M \times \exp\left(-\frac{1}{2\pi\alpha'}\int d^2\sigma\,\sqrt{g}g^{\alpha\beta}\partial_\alpha x^M\partial_\beta x^N G_{MN}\right)|_{\{x_1,x_2,x_3,x_4\}} \quad (10)$$

with $M, N = 0, \dots, D$ ($D = 9$ for the superstring). The above proposal (10) is conjectured to hold for all gauge/gravity dual pairs. The information about the specific gauge theory is encoded in the metric G_{MN}. We will use Witten's model of back-reacted N_c D4 branes compactified on a thermal circle [10]. The conjecture is [10] that type IIA string theory on

$$ds^2 = (U/R)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2) + (R/U)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$
$$f(U) = 1 - U_{KK}^3/U^3\,, \quad (11)$$

with anti-periodic condition for NS-R modes on τ is dual to large-N Yang–Mills theory. Unfortunately, in this model the scale of the Kaluza–Klein modes and the scale of confinement are of the same order, $\Sigma \sim (g^2N_c)M_{KK}^2$ [11]. Note that since we use the type IIA superstring, we should use in (10) the Polyakov action for the superstring, namely we should incorporate worldsheet fermions.

A calculation of Wilson loop expectation value in Witten's background yields an area law for large loops, $\langle W\rangle = \exp(-\Sigma A)$, with a string tension $\Sigma = \frac{1}{2\pi\alpha'}(\frac{U_{KK}}{R})^{\frac{3}{2}}$. The reason is that for a sufficiently large loop the string will quickly drop to $U = U_{KK}$ and the main contribution to the action is essentially by a flat space worldsheet that resides close to U_{KK} [12].

It is therefore anticipated that if the points $\{x_1, x_2, x_3, x_4\}$ are well separated from each other $|x_i - x_j| \gg 1/\sqrt{\Sigma}$, the sum over string worldsheets will be dominated by flat space configurations, as depicted in Fig. 3. This claim is supported by the analysis of Ref. [6], where the authors showed that under the assumption of confinement (area law) the sum over Wilson loops is dominated by a saddle-point that corresponds to the IR piece of the worldsheet in Fig. 3. Our claim is indeed similar to that of Ref. [6]: assuming that Wilson loop expectation values are calculated by a flat space string configuration implies that they admit an area law.

String worldsheets that span in the compact directions (τ and the four-sphere) are expected to admit a larger action than string worldsheets that reside in the flat four dimensional space.