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High temperature dimensional reduction in Snyder space

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ABSTRACT

In this paper, we formulate the statistical mechanics in Snyder space that supports the existence of a minimal length scale. We obtain the corresponding invariant Liouville volume which properly determines the number of microstates in the semiclassical regime. The results show that the number of accessible microstates drastically reduces at the high energy regime such that there is only one degree of freedom for a particle. Using the Liouville volume, we obtain the deformed partition function and we then study the thermodynamical properties of the ideal gas in this setup. Invoking the equipartition theorem, we show that 2/3 of the degrees of freedom freeze at the high temperature regime when the thermal de Broglie wavelength becomes of the order of the Planck length. This reduction of the number of degrees of freedom suggests an effective dimensional reduction of the space from 3 to 1 at the Planck scale.

1. Introduction

While general relativity and quantum mechanics are successful in their applicability domains, it seems that there is a fundamental incompatibility between them in order to find the so-called quantum theory of gravity. Such a theory, not completely formulated so far, would reasonably describe the structure of spacetime at the Planck scale where both of the gravitational and quantum mechanical effects become important. Despite the fact that there is no unique approach to quantum gravity, existence of a universal minimum measurable length, preferably of the order of the Planck length $l_{\rm Pl} \sim 10^{-33}$ m, is a common feature of quantum gravity candidates such as string theory and loop quantum gravity [1,2]. It is then widely believed that a non-gravitational theory which includes a universal minimal length scale would appear at the flat limit of quantum gravity. Therefore, many attempts have been done in order to take into account a minimal length scale in the well-known non-gravitational theories such as quantum mechanics and special relativity. The generalized uncertainty principle is investigated in the context of the string theory that supports the existence of a minimal length as a nonzero uncertainty in position measurement [3]. Quantum field theories turn out to be naturally ultraviolet-regularized in this setup [4]. Inspired by the seminal work of Snyder in 1947 who was formulated a Lorentz-invariant noncommutative spacetime [5], a phase space with noncanonical symplectic structure is formulated in the non-relativistic limit [6]. At the quantum level, this deformed phase space leads to the modified uncertainty relation which is very similar to one arises from the string theory motivations [6]. Furthermore, recently, polymer quantum mechanics was suggested in the symmetric sector of loop quantum gravity which supports the existence of a minimal length scale known as the polymer length scale [7]. Also, the doubly special relativity theories are investigated in order to take into account a minimal observer-independent length scale in special relativity [8]. Appearance of curved energy–momentum space is the direct consequence of the doubly special relativity theories [9] and, interestingly, the Snyder noncommutative spacetime could be also realized in this setup by a relevant gauge fixing process [10].

Apart from the details of the above mentioned phenomenological models as a flat limit for quantum gravity, all of them suggest the deformation to the density of states at the high energy regime which in turn leads to the nonuniform measure over the set of microstates. Indeed, in these setups, the number of accessible microstates will be reduced at the high energy regime due to the existence of a minimal length as an ultraviolet cutoff for the system under consideration. Reduction of the number of degrees of freedom, however, immediately suggests an effective dimensional reduction of the space. This consequence seems to be a general feature of quantum gravity which may also open new window for the statistical origin of black holes thermodynamics [11]. Thermodynamics of black holes is widely studied in the frameworks of phenomenological quantum gravity models such as noncommutative space [12], generalized uncertainty principle [13] and polymer

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quantization scenario [14]. The reduction of the number of accessible microstates due to the universal quantum gravitational effects would also significantly change the thermodynamical properties of any physical system at the high temperature regime. Therefore, quantum gravity effects on the thermodynamics of various statistical systems are widely studied in different contexts [15]. For the special case of the ideal gas, it is natural to expect that the quantum gravity effects would become important at the high temperature regime, when the corresponding thermal de Broglie wavelength $\lambda = \sqrt{\frac{2\pi}{mT}}\,\hbar$ becomes of the order of the Planck length $l_{\rm Pl} = \sqrt{\hbar G}$, where m is the particles' mass and T denotes the temperature.[1] The associated thermodynamical properties then will be significantly modified in this regime. Thermodynamics of the ideal quantum gases in noncommutative space is studied in Refs. [16,17] and for the case of the effects that arise from the generalized uncertainty principle see Ref. [18]. Thermodynamical properties of the ideal gas in polymerized phase space, as a classical limit of a polymer quantum mechanics, are also studied in Refs. [19–21]. For the case of the relativistic ideal gases in doubly special relativity framework see Refs. [22,23]. Motivated by the above stated issues, in this paper we study the thermodynamical properties of the ideal gas in Snyder space.

The structure of the paper is as follows: In Section 2, the statistical mechanics in the Snyder space is formulated and the corresponding partition function is found. Using the partition function, thermodynamics of the ideal gas is studied in Section 3. Section 4 is devoted to the summary and conclusions.

2. Statistical mechanics in Snyder space

The kinematics and dynamics of a classical system on the phase space provide a suitable framework for formulating the statistical mechanics in the semiclassical regime. The key quantity is the Liouville volume that determines the density of states from which all the thermodynamical properties of a system could be achieved. In this section, using the symplectic geometry, we formulate the statistical mechanics in Snyder space.

2.1. Kinematics and dynamics

Inspired by the seminal work of Snyder on noncommutative spacetime [5], the associated deformed phase space is formulated which also supports the existence of a minimal length [6]. A phase space naturally admits symplectic structure and therefore is a symplectic manifold. Suppose that (Γ, ω) to be a Snyder-deformed phase space with ω as the associated symplectic structure which is a closed nondegenerate 2-form on Γ. The local form of the symplectic structure in Snyder model is given by [24]

$$\omega = dq^i \wedge dp_i - \frac{1}{2}\, d(q^i p_i) \wedge d\ln\left[1+\beta^2 p^2\right], \tag{1}$$

where q^i and p_i are the position and momentum coordinates of a particle with $i, j = 1, \ldots, 3$ and $p^2 = \delta^{ij} p_i p_j$. β is the deformation parameter with dimension of length which is usually taken to be of the order of the Planck length as $\beta = \beta_0\, l_{\rm Pl}$, where $\beta_0 = \mathcal{O}(1)$ is the dimensionless numerical constant that should be fixed only with experiment [25]. Taking the low energy limit $\beta \to 0$ in the relation (1), the standard well-known canonical form of the symplectic structure could be recovered.

Since the symplectic structure is nondegenerate by definition, one can assign a unique vector field $\mathbf{x}_f$ to any function f on Γ as $\omega(\mathbf{x}_f) = df$. The Poisson bracket for two real-valued functions is defined as

$$\{f, g\} = \omega(\mathbf{x}_f, \mathbf{x}_g)\,. \tag{2}$$

From the above definition, it is straightforward to show that the symplectic structure (1) generates the following noncanonical Poisson algebra

$$\{q^i, q^j\} = \beta^2 J^{ij}, \qquad \{q^i, p_j\} = \delta^i_j + \beta^2 p^i p_j, \qquad \{p_i, p_j\} = 0, \tag{3}$$

where $J_{ij} = q_i p_j - q_j p_i$ is the generator of the rotation group in three dimensions with $q_i = \delta_{ij} q^j$. The symplectic structure (1) or equivalently the Poisson algebra (3) properly defines the kinematics of the phase space Γ in Snyder model.

The dynamics of the system will be determined by specifying a Hamiltonian function H as the generator of time evolution of the system. The Hamiltonian system on the phase space is then defined by the triplet (Γ, ω, H) and the dynamical evolution of the system is governed by the equation

$$\omega(\mathbf{x}_H) = dH\,, \tag{4}$$

where $\mathbf{x}_H$ is the Hamiltonian vector field and it's integral curves are nothing but the Hamilton's equations in this setup (see Ref. [24] for more details).

Furthermore, the natural volume on the phase space is the Liouville volume which for a $2n$-dimensional phase space is defined as

$$\omega^n = \frac{1}{n!}\, \omega \wedge \ldots \wedge \omega \qquad (n \text{ times})\,. \tag{5}$$

The Liouville volume for a particle in Snyder-deformed phase space then could be easily obtained by substituting the symplectic structure (1) into the definition (5) which gives

$$\omega^3 = dq^1 \wedge dq^2 \wedge dq^3 \wedge \frac{dp_1 \wedge dp_2 \wedge dp_3}{\left(1+\beta^2 p^2\right)}\,. \tag{6}$$

The phase space (Liouville) volume determines the density of states and then the number of accessible microstates for a statistical system. It is important to check the verification of the Liouville theorem for the Snyder measure (6) in order to formulate the statistical mechanics in Snyder-deformed phase space. The Liouville theorem states that the Liouville volume is invariant under the time evolution of the system

$$\frac{d\omega^n}{dt} = \frac{\partial \omega^n}{\partial t} + \mathcal{L}_{\mathbf{x}_H} \omega^n = 0\,, \tag{7}$$

where $\mathcal{L}_{\mathbf{x}_H}$ denotes the Lie derivative with respect to $\mathbf{x}_H$. The relation (7) can be traced back to the facts that ω^n is not explicitly time-dependent, $\mathcal{L}_{\mathbf{x}_H}\omega^n = n(\mathcal{L}_{\mathbf{x}_H}\omega) \wedge \omega^{n-1}$ and $\mathcal{L}_{\mathbf{x}_H}\omega = d(\omega(\mathbf{x}_H)) + (d\omega)(\mathbf{x}_H) = 0$, where we have used the equation (4) as $d(\omega(\mathbf{x}_H)) = d^2 H = 0$ and the closure of the symplectic structure $d\omega = 0$. The result (7) for the Snyder measure (6) is essential for us to formulate the statistical mechanics in Snyder space.

2.2. Number of microstates

Before obtaining the partition function, from which all the thermodynamical properties of a system can be obtained, we first make a qualitative discussion about the number of microstates in Snyder-deformed phase space regardless of the ensemble density and the Hamiltonian function.

The number of microstates for a single-particle phase space is given by

[1] We work in units $k_B = 1 = c$, where k_B and c are the Boltzmann constant and speed of light in vacuum respectively.