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Scattering of Klein–Gordon particles by a Kink-like potential

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HIGHLIGHTS

- The Klein–Gordon equation for the non-minimal vector and a scalar Kink-like potential is solved.
- We have calculated the corresponding transmission and reflection coefficients.
- We discussed the behavior of the reflection and transmission coefficients vs. energy.

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ABSTRACT

The Klein–Gordon equation for the non-minimal vector and a scalar Kink-like potential is solved in terms of the hypergeometric functions. The scattering problem, i.e. the transmission and reflection coefficients, is studied as well.

1. Introduction

The Klein–Gordon (KG) equation (KGE), because of describing relativistic spin-zero particles, has been the subject of various studies in different physical sciences. Consequently, possible analytical

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techniques have been applied to the problem to obtain exact or approximate solutions of the KGE for various interactions which yield successful phenomenological outcomes [1–9]. A particular case of interest is the scattering of the wave function which is an essential primary step in many fields. The scattering problem has been investigated for both the nonrelativistic and relativistic regimes via the Schrödinger [10–13], KG [14–16] and Dirac equations [17–20]. The main purpose of this article is to investigate the time-independent Klein–Gordon equation with vector and scalar Kink-like potentials in $(1+1)$ dimensions for spin-zero bosons via the KGE. This potential has been the subject of some researches. The scalar and vector terms of a Kink-like potential is given in the following form

$$\begin{aligned} V(x) &= V_0\{\Theta(-x)[1-\tanh(-\alpha x)]+\Theta(x)[1-\tanh(\alpha x)]\} \\ &= V_0\left[\Theta(-x)\left(\frac{2e^{2\alpha x}}{1+e^{2\alpha x}}\right)+\Theta(x)\left(\frac{2e^{-2\alpha x}}{1+e^{-2\alpha x}}\right)\right], \end{aligned} \tag{1a}$$

$$\begin{aligned} S(x) &= V_1[\Theta(-x)[1-\tanh(-\alpha x)]+\Theta(x)[1-\tanh(\alpha x)]] \\ &= V_1\left[\Theta(-x)\left(\frac{2e^{2\alpha x}}{1+e^{2\alpha x}}\right)+\Theta(x)\left(\frac{2e^{-2\alpha x}}{1+e^{-2\alpha x}}\right)\right], \end{aligned} \tag{1b}$$

where $\Theta(x)$ is the Heaviside step function and V_0, V_1 are the strength of the potential. Kink-like potentials arise in quantum field theory, and more precisely, the study of topological classical backgrounds [21,22]. Trapping of Neutral fermions with such a potential was investigated by de Castro and Hott by mapping the problem into a Strum–Liouville equation [23]. The evolution of the particle spectrum of two-parameter families of non-integrable quantum field theories was studied in [24]. Chiral states in bilayer graphene were the subject of the research of Zarenia et al. by considering a kink–antikink potential [25]. Exact solutions of the KGE with position-dependent mass for mixed vector and scalar kink-like potentials were obtained by Jia et al. in Ref. [26]. Our paper is organized as follows. In Section 2, we introduce the Klein–Gordon equation in $(1+1)$ dimensional space–time. Next, we obtain the exact solution of the KGE in terms of hypergeometric functions. In Section 3, the useful transmission and reflection coefficients are calculated.

2. The radial Klein–Gordon equation in one-dimension with the Kink-like potential

We already know that the one-dimensional time-independent KGE can be written as [5]

$$\psi''(x)+\frac{1}{\hbar^2c^2}\left[[E-V(x)]^2-\left[mc^2+S(x)\right]^2\right]\psi(x)=0, \tag{2}$$

where $V(x)$, $S(x)$ and E are vector, scalar potentials and the energy of the relativistic particle respectively. As we are searching for the scattering states of the equation for a Kink-like potential barrier, we study the wave functions for $x<0$. From substitution of Eqs. (1a) and (1b) into Eq. (2), we have

$$\frac{d^2\psi_L(x)}{dx^2}+\frac{1}{\hbar^2c^2}\left\{\left[E-V_0\left(\frac{2e^{2\alpha x}}{1+e^{2\alpha x}}\right)\right]^2-\left[mc^2+V_1\left(\frac{2e^{2\alpha x}}{1+e^{2\alpha x}}\right)\right]^2\right\}\psi_L(x)=0, \tag{3}$$

or

$$\begin{aligned} &\frac{d^2\psi_L(x)}{dx^2}+\frac{1}{\hbar^2c^2}\left\{\frac{1}{(1+e^{2\alpha x})^2}[4V_0^2e^{4\alpha x}-4V_1^2e^{4\alpha x}]+\frac{1}{(1+e^{2\alpha x})}[-4EV_0e^{2\alpha x}\right. \\ &\left.\quad -4mc^2V_1e^{2\alpha x}]+E^2-m^2c^4\right\}\psi_L(x)=0 \end{aligned} \tag{4}$$

by applying the new variable $z=-e^{2\alpha x}$, Eq. (4) appears as

$$z(1-z)\frac{d^2\psi_L(z)}{dz^2}+(1-z)\frac{d\psi_L(z)}{dz}+\frac{1}{z(1-z)}\{\omega_1z^2+\omega_2z+\omega_3\}\psi_L(z)=0, \tag{5}$$