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Critical coupling and coherent perfect absorption for ranges of energies due to a complex gain and loss symmetric system

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HIGHLIGHTS

- Energy ranges for CC & CPA are obtained explicitly for complex WS potential.
- Analytical conditions for CC and CPA for PT symmetric WS potential are obtained.
- Conditions for left & right CC are shown to be different.
- Conditions for CC & CPA are shown to be that of SS for the time reversed system.
- Our model shows the great flexibility of frequencies for CC & CPA.

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ABSTRACT

We consider a non-Hermitian medium with a gain and loss symmetric, exponentially damped potential distribution to demonstrate different scattering features analytically. The condition for critical coupling (CC) for unidirectional wave and coherent perfect absorption (CPA) for bidirectional waves are obtained analytically for this system. The energy points at which total absorption occurs are shown to be the spectral singular points for the time reversed system. The possible energies at which CC occurs for left and right incidence are different. We further obtain periodic intervals with increasing periodicity of energy for CC and CPA to occur in this system.

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1. Introduction

The recent ideas of PT-symmetric non-Hermitian quantum mechanics [1–3] have been fruitfully extended to optics due to formal equivalence between Schrödinger equation and certain wave equations in optics [4–6]. The parity operator P stands for spatial reflections ($x \rightarrow -x, p \rightarrow -p$), while the anti-linear time reversal operator T leads to ($i \rightarrow -i, p \rightarrow -p, x \rightarrow x$). The equivalence between quantum mechanics and optics becomes possible due to the judicious inclusion of complex refractive index distribution $V(x) = \eta_R(x) + i\eta_I(x)$, in the electromagnetic wave equation [5,7]. To realize this consider a one dimensional optical system with effective refractive index $\eta_R(x) + i\eta_I(x)$ in the background of constant refractive index η_0, η_I stands for gain and loss component. The electric field $E = E_0(x,z)e^{i(wt-kz)}$ of a light wave propagating in this medium (with $\eta_0 \gg \eta_I, \eta_R$) satisfies the Schrödinger like equation,

$$\begin{aligned} i\frac{\partial}{\partial z}E_0(x,z) &= \left[\frac{1}{2k}\frac{\partial^2}{\partial x^2} + k_0[\eta_R(x) + i\eta_I(x)]\right]E_0(x,z) \\ &= HE_0(x,z) \end{aligned} \tag{1}$$

$k = k_0\eta_0$, k_0 being the wave number in vacuum. This Hamiltonian H has gain and loss symmetry for $\eta_R(x) = \eta_R(-x)$ and $\eta_I(x) = -\eta_I(-x)$. Thus complex optical gain and loss potential can be realized by judiciously incorporating gain and loss profile on an even index distribution. This important realization opens a wide window to study optical systems with gain and loss medium and leads to the experimental observation of PT-invariance and its breaking [8–17] in various optical systems [4–7,18,19].

Various features of quantum scattering due to non-Hermitian potentials like exceptional points [20–22] and spectral singularity (SS) [23–25], reflectionlessness and invisibility [26–29], reciprocity [26,25,30] etc. have generated huge interest due to their applicability and usefulness in the study of optics. Recently the observation of perfect absorption [31–44] of incident electromagnetic wave by an optical media with complex refractive index distribution is counted as a big achievement in optics. The coherent perfect absorber (CPA) which is actually the time-reversed counterpart of a laser has become the center to all such studies in optics due to the discovery of anti-laser [31–33] in which incoming beams of light interfere with one another in such a way as to perfectly cancel each other out. This phenomena of perfect absorption in optics can also be observed in quantum scattering when particles (with a mass m and total energy E) interact with the surrounding medium through a complex potential distribution $V(x)$.

Scattering due to complex potential can be described in a simple mathematical language as follows. If A and B are the incident wave amplitudes from left and right directions and C and D are the outgoing wave amplitudes to left and right respectively, then the scattering amplitudes are related through scattering matrix as,

$$\begin{pmatrix} C \\ D \end{pmatrix} = S\begin{pmatrix} A \\ B \end{pmatrix}, \quad \text{where } S = \begin{pmatrix} t_l & r_r \\ r_l & t_r \end{pmatrix}. \tag{2}$$

For the perfect absorption the outgoing amplitudes C and D vanish leading to,

$$t_lA + r_rB = 0; \qquad r_lA + t_rB = 0; \tag{3}$$

The condition for perfect absorption for unidirectional incident waves can be written as,

$$\begin{aligned} &t_l = 0;\ r_l = 0, \quad \text{for left incidence } (B = 0) \\ &t_r = 0;\ r_r = 0, \quad \text{for right incidence } (A = 0) \end{aligned} \tag{4}$$

These situations are known in literature as critical coupling (CC) [40–44]. On the other hand for bidirectional incident wave the condition for perfect absorption is,

$$|\det[S]| = |t_lt_r - r_lr_r| = 0 \tag{5}$$

This condition refers as coherent perfect absorption which has recently created lots of excitements [31–39] due to the discovery of anti-laser. This could pave the way for a number of novel technologies with various applications from optical computers to radiology [31,33]. Thus it is extremely important to investigate different aspects of CPA using different non-Hermitian systems.