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Wave packet dynamics in various two-dimensional systems: A unified description

Ashutosh Singh, Tutul Biswas *, Tarun Kanti Ghosh, Amit Agarwal

Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India

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ABSTRACT

In this article we present an exact and unified description of wave packet dynamics in various 2D systems in the presence of a transverse magnetic field. We consider an initial minimum-uncertainty Gaussian wave packet and find that its long-term dynamics displays the universal phenomena of spontaneous collapse and quantum revival. We estimate the timescales associated with these phenomena based on very general arguments for various materials, whose carrier dynamics is described either by the Schrödinger equation or by the Dirac equation.

1. Introduction

Spontaneous collapse and consequent quantum revival [1,2] occurs in the long-term dynamics of an injected wave packet in systems with non-equidistant energy levels due to quantum interference. It has been investigated in a wide class of systems [3] and the phenomenon of wave packet collapse, revivals, and fractional revivals have been observed experimentally in a number of atomic and molecular systems [4–7]. However this phenomenon of purely quantum mechanical in origin is relatively less explored in condensed matter systems with discrete Landau energy levels, despite the fact that unlike *zitterbewegung* in solid state systems [8–10] these oscillations are large and slow enough for an experimental probe.

Zitterbewegung and wave packet dynamics in several 2D condensed matter systems with Landau-levels have been explored earlier [10–16], but the phenomena of spontaneous collapse and revival

* Corresponding author.
E-mail address: tutulb@iitk.ac.in (T. Biswas).

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have largely gone unaddressed [17–19]. Motivated by the unified description of *zitterbewegung* in solid state systems in Refs. [20,21], in this article we present an exact and unified description of the quantum wave packet dynamics and the phenomena of collapse and quantum revival in various two dimensional (2D) solid state systems. Initially when a well-localized wave packet is injected into a 2D system with non-equidistant Landau energy levels, it undergoes a cyclotron motion and evolves quasi-classically with periodicity $\tau_{\rm cl}$, for a number of cycles, with its probability density spreading around the quasi-classical trajectory. Non-equidistant nature of the discrete energy spectrum then leads to destructive quantum interference and consequently the collapse of the wave packet. The (almost) collapsed wave packets regain their initial waveform and oscillate again with the quasi-classical periodicity on a much longer time scale known as revival time ($\tau_{\rm rev} \gg \tau_{\rm cl}$). In addition, there is also the possibility of fractional revivals which occurs at rational fraction of the revival time $\tau_{\rm rev}$ when the initial wave packet evolves into a collection of mini wave packets resembling the waveform of the injected wave packet [2,3].

In this article we consider several 2D systems, which can be classified into two categories based on the dynamics of the charge carriers: Schrödinger-like (non-relativistic dynamics described by the Schrödinger equation) and Dirac-like fermionic systems [22] (relativistic dynamics described by the Dirac equation). The so-called Schrödinger-like materials include 2D electron/hole gas (2DEG/2DHG) trapped at the interface of III–V semiconductor hetero-structures, like AlGaAs–GaAs, and these typically have a linear [23,24] or cubic spin–orbit interaction (SOI) terms [25–30] in addition to the parabolic dispersion relation. The so-called Dirac-like materials have a relativistic dispersion relation and typical examples include graphene [31–33] and other crystals like silicene [34–38], germanene [39,40], monolayer transition metal group-VI dichalcogenides MX_2 ($M = Mo$, W and $X = S$, Se) etc. [41–44] which generally have a honeycomb lattice structure. Dirac materials also have suppressed electron scattering and tunable electronic properties which make them very interesting from an application point of view.

For the present study, we consider both class of systems on an equal footing and present a unified and an exact description of quantum wave packet dynamics whose long-term behavior displays the universal phenomena of spontaneous collapse and revival. For this purpose we choose the initial localized wave packet to be a coherent state, which is also a minimum uncertainty wave packet, whose cyclotron dynamics resembles the dynamics of a classical charged particle in a perpendicular magnetic field [11].

Our article is organized as follows: in Section 2 we study the wave packet dynamics in an exact and unified manner for various 2D systems with Landau levels. In addition we motivate and discuss the timescales associated with the phenomena of spontaneous collapse and revival in systems with discrete and non-equidistant energy levels. In Section 3, we discuss the collapse and revival phenomenon in Schrödinger-like materials with parabolic energy spectrum and k-linear and k-cubic Rashba SOI. In Section 4 we discuss wave packet dynamics in Dirac-like materials with a relativistic dispersion, and finally, in Section 5 we summarize our results.

2. Unified description of wave packet dynamics in various 2D systems

In this section we present an exact and unified formalism describing the temporal evolution of a wave packet in various 2D systems, in the presence of a transverse magnetic field. In particular, we focus on both Schrödinger-like systems and Dirac-like materials, whose low energy properties are described by a two band model. In Section 2.1 we calculate the exact expectation value of the position and velocity operator (or alternatively electric current), for an injected coherent state minimum uncertainty wave packet in generic 2D systems. Next in Section 2.2, we briefly review the phenomenon of wave packet revival in various 2D systems with Landau level spectrum.

2.1. Exact quantum evolution of a wave packet in various 2D systems

We begin by presenting an exact unified description of the temporal evolution of the center of an injected wave packet in various two dimensional systems with non-equidistant Landau energy levels.