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Entropy of massive quantum fields in de Sitter space–time

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ABSTRACT

Using the quantum states or Hilbert spaces for the quantum field theory in de Sitter ambient space formalism the entropy of the massive quantum field theory is calculated. In this formalism, the homogeneous spaces which are used for construction of the unitary irreducible representation of de Sitter group are compact. The unique feature of this homogeneous space is that by imposing certain physical conditions its total number of quantum one-particle states, $\mathcal{N}_{1-p}$, becomes finite although the Hilbert space has infinite dimensions. $\mathcal{N}_{1-p}$ is de Sitter invariant and a continuous function of the Hubble constant H and the eigenvalue of the Casimir operators of de Sitter group. The entropy of the quantum fields is finite and invariant for all inertial observers on de Sitter hyperboloid.

1. Introduction

An inertial observer in the de Sitter (dS) vacuum state detects a thermal radiation at the temperature $T = \frac{H}{2\pi}$ for each coordinate system on the dS hyperboloid. Its corresponding thermodynamic entropy is [1]

$$S_{\text{dS}} = \frac{A_c}{4G} = \frac{\pi}{GH^2} = \frac{1}{4\pi GT^2}, \tag{1.1}$$

where G is Newton's constant and $A_c = 4\pi H^{-2}$ is the area of the cosmological event horizon in dS space. This entropy is similar to the Bekenstein–Hawking entropy of the black hole: $S_{BH} = \frac{k_B c^3}{\hbar}\frac{A_h}{4G}$, where A_h is the area of black hole horizon, c is the speed of light, k_B is the Boltzmann constant and

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$\hbar = \frac{h}{2\pi}$ is the Planck constant. The microscopic origin of this entropy is an outstanding problem in theoretical physics [2–5]. The structure of the Hilbert space of the physical system, which includes the gravitational field is unknown mostly because of the lack of a proper method to quantize the gravity. Also, the Hilbert space of quantum field theory in curved space–time has infinite dimension. Then the total number of the quantum states is infinite and a finite entropy cannot be obtained. These problems have been surveyed in different methods by several authors [3–18].

In this paper, the microscopic origin of the dS entropy, in the context of quantum field theory (QFT) in dS ambient space formalism is considered by using the calculation of the total number of quantum states. For this purpose the Hilbert spaces of QFT in dS ambient space formalism are recalled [19]. The UIR of the dS group and their corresponding Hilbert spaces are constructed on compact homogeneous spaces with the finite total volume. Each point in these homogeneous spaces represents a vector in the Hilbert spaces which is infinite mathematically. Since an even horizon or a maximum length for an observable in dS space exists then a minimum size in the compact Homogeneous space can be defined by using the Heisenberg uncertainty principle. Since the total volume of Homogeneous space is finite and a minimum length in this space exists from uncertainty principle, therefore the total number of points becomes finite physically. The total number of the quantum one-particle states, $\mathcal{N}_{1-p}$, is the integration over the admissible value of the points in the homogeneous space and for the compact homogeneous space it is finite. The total number of quantum states on Fock space can be easily renormalized and obtain a finite value $\mathcal{N}_{ren}$.

Since the Hilbert space is constructed on a homogeneous space, which is invariant under the scale transformation, then an arbitrary scale appears in the total number of quantum one-particle state $\mathcal{N}_{1-p}$. $\mathcal{N}_{1-p}$ is an arbitrary function of the Hubble parameter and the eigenvalues of the Casimir operators of the dS group. This arbitrariness can be fixed by imposing some physical conditions such as the Holographic principle or the entropy bounds [20–32]. Finally, the entropy of the massive quantum fields is calculated, which is finite. It is a function of the Hubble parameter and the eigenvalues of the Casimir operators of the dS group. These parameters are the dS invariant and equivalent for each inertial observer.

2. Number of quantum states

The dS space–time can be identified by a 4-dimensional hyperboloid embedded in 5-dimensional Minkowskian space–time with the constraint:

$$M_H = \{x \in \mathbb{R}^5; x^2 = \eta_{\alpha\beta}x^\alpha x^\beta = -H^{-2}\}, \quad \alpha, \beta = 0, 1, 2, 3, 4, \tag{2.1}$$

where $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1, -1)$. The dS metrics is

$$ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta|_{x^2=-H^{-2}} = g^{\text{dS}}_{\mu\nu}dX^\mu dX^\nu, \quad \mu = 0, 1, 2, 3, \tag{2.2}$$

where X^μ are 4 space–time intrinsic coordinates system on dS hyperboloid.

The UIR of Poincaré group in energy–momentum space can be defined as [33]

$$\mathcal{P}^{(M,j)}(\Lambda, a)\left|k^\mu, m_j; j, M\right\rangle = e^{-ia.\Lambda k}\sqrt{\frac{(\Lambda k)^0}{k^0}}\sum_{m'_j} D^{(j)}_{m'_j m_j}(W(\Lambda, k))\left|(\Lambda k)^\mu, m'_j; j, M\right\rangle, \tag{2.3}$$

where $\Lambda \in SO(1, 3)$, $a^\mu \in \mathbb{R}^4$ and $D^{(j)}_{m'_j m_j}(W(\Lambda, k))$ furnishes a certain representation of $SU(2)$ group, which is defined explicitly in [33]. The two parameters j and M are classifying the UIR of the Poincaré group and determining the eigenvalues of the two Casimir operators of this group. The infinite dimensional one-particle Hilbert space $\mathcal{H}_k^{(j,M)}$ is given by:

$$\left|k^\mu, m_j; j, M\right\rangle \in \mathcal{H}_k^{(j,M)}, \qquad (k^0)^2 - (\vec{k}.\vec{k}) = M^2, \quad -j \leq m_j \leq j.$$

In the Minkowskian space for the free particle plane wave, in order to count the number of states it is convenient to use the box normalization convention, a cubic box of side L. The maximum length L results in a minimum size for momentum $(k)_{\min} \sim L^{-1}$ ($\vec{k} = \frac{2\pi}{L}\vec{n}$). Then the total number of quantum