C. R. Physique 16 (2015) 10–25

Contents lists available at ScienceDirect

Comptes Rendus Physique

www.sciencedirect.com

Granular physics / Physique des milieux granulaires

Jamming in granular materials

Le blocage des matériaux granulaires

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A R T I C L E   I N F O

Article history:
Available online 20 February 2015

Keywords:
Granular materials
Jamming
Shear jamming
Impact
Force networks

Mots-clés :
Matériaux granulaires
Blocage
Blocage au cisaillement
Impact
Chaînes de force

A B S T R A C T

Granular materials are one of a class of materials which undergo a transition from mechanically unstable to mechanically stable states as key system parameters change. Pioneering work by Liu and Nagel and O'Hern et al. focused on models consisting of frictionless grains. In this case, density, commonly expressed in terms of the packing fraction, ϕ, is of particular importance. For instance, O'Hern et al. found that there is a minimum $\phi = \phi_J$, such that below this value there are no jammed states, and that above this value, all stress-isotropic states are jammed. Recently, simulations and experiments have explored the case of grains with friction. This case is more subtle, and ϕ does not play such a simple role. Recently, several experiments have shown that there exists a range of relatively low ϕ's such that at the same ϕ it is possible to have jammed, unjammed, and fragile states in the sense of Cates et al. This review discusses some of this recent work, and contrasts the cases of jamming for frictionless and frictional granular systems.

R É S U M É

Les matériaux granulaires appartiennent à une classe de matériaux qui subissent une transition d'instabilité mécanique à stabilité tandis que les paramètres du système changent. Les travaux novateurs de Liu et Nagel et O'Hern et al. ont considéré des grains sans frottement. Dans ce cas, la densité, qui est caractérisée comme d'habitude par la fraction volumique des solides ϕ, revêt une importance particulière. Par exemple, O'Hern et al. ont trouvé qu'il existe un minimum $\phi = \phi_J$ tel que, en dessous de cette valeur, il n'existe pas de blocage et que, au-dessus, tous les états où la contrainte est isotropique sont bloqués. Recemment, des simulations et des expériences ont exploré le cas de grains frottants, qui est à la fois plus subtil et où ϕ n'a plus un rôle aussi simple. Dernièrement, plusieurs expériences ont montré qu'il existe un domaine de petites valeurs de ϕ où coexistent des états bloqués, libres ou fragiles au sens de Cates et al. Cette revue aborde une partie de ces récents travaux, et compare les cas de blocage pour les systèmes granulaires frottants ou non.

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http://dx.doi.org/10.1016/j.crhy.2015.02.001

1. Introduction

The concept of jamming, which characterizes how disordered particle systems transition from fluid-like to solid-like states, has attracted significant recent interest, in no small part due to the fact that a range of different systems undergo a jamming transition and/or exhibit glassy behavior. In some cases, the particles forming the system are truly macroscopic, as in the case of foams and granular materials. Smaller particles can occur in emulsions and colloids, and of course molecular glasses are truly microscopic. All these systems have certain features in common, such as spatial disorder and hindered motion when they are densely packed and "cold". Several reviews of jamming are available [1–3], as well as extensive reviews of glassy systems [4].

Until recently, it was thought that systems as diverse as colloids, foams, emulsions and granular materials could be unified into a single picture through a jamming diagram, involving thermodynamic temperature, T, packing fraction, ϕ, and shear stress, τ, as proposed by Liu and Nagel [5]. This proposal was intensively investigated, chiefly by numerical simulations [6–9] that modeled frictionless particles interacting via normal contact forces. Such systems can be realized experimentally as foams [3] or emulsions [10]. In this case, there is a lowest packing fraction, ϕ_J, which may depend on system preparation, below which states are always unjammed, and above which jammed states exist for a range of shear stresses, $0 \le \tau < \tau_Y$, where $\tau_Y(\phi)$ is the yield stress. Much of that work has been reviewed recently [1–3].

The present review focuses on the nature of jamming in a particular class of macroscopic particle systems, granular materials. This term has been used broadly to label collections of macroscopic particles for which there is no mutual exchange of energy with a heat bath. For granular materials, dissipation during particle interactions injects heat into the surroundings, but the motion of grains is not affected in any substantive way by thermally heating them.

Everyday materials such as sand, salt, food grains, pharmaceutical powders or lumps of coal all qualify as 'granular materials'. All of these systems interact by contact forces: grains interact only when their surfaces are in contact, and the interaction forces are either normal to the plane of contact, or in the plane of contact, due to friction. Typically, the repulsive normal forces involve elastic deformation of the contacting particles and are conservative for slow strains, but not for fast strains. Some granular models only use conservative normal forces, i.e. the model particles are frictionless. The characteristics of jamming for frictionless and frictional particles differ in several respects, including relatively simple shifts of parameters at jamming. However, there are also much more significant differences in the types of near-jamming states for the case of frictional vs. frictionless particles.

Mechanical stability and response: At a naive level, the idea of jamming is simple: typically, disordered solid-like particle systems respond to applied strains by deforming, but without flowing; thus, jammed systems have non-zero compressional and shear moduli. Unjammed systems respond to shear strain by flowing, although, damping, e.g., viscosity or friction, may act to limit the flow.

A necessary condition for a system of particles to be jammed is that particles are mechanically stable, i.e. that they are in force and torque balance. If a system of particles is in a mechanically stable state, it is generally referred to as jammed. There may be a small set of particles, called rattlers, which are not subject to any forces. Rattlers can be removed without otherwise affecting the system. There also exist states which satisfy force and torque balance, but are not stable to particular small strains. These are fragile states, and they were first proposed by Cates et al. [11] in the context of sheared colloidal systems.

A key parameter which controls whether a system is jammed or not is the average number of contacts per particle, Z. One way to understand the role of Z is to count degrees of freedom associated with vector forces at contacts, and constraints associated with force and torque balance. For example, a system of N frictionless spheres in dimension d has $NZ/2$ independent forces, and dN force balance constraints. (Note that by Newton's third law, there are half as many independent forces as there are force-bearing contacts.) Thus, for such a system to be mechanically stable, there must be at least $Z = 2d$ contacts on average per particle. A system can have more contacts and be stable, but not less. The marginal or isostatic state exactly satisfies $Z_{iso} = 2d$. A similar argument for 'ideal' frictional particles (i.e. large static friction coefficient, μ) yields an isostatic condition, $Z_{iso} = d + 1$. In fact, numerical simulations [12,13] for $d = 2$ show that Z at jamming depends on μ, with a smooth variation of Z_{iso} between the frictionless case, where $Z_{iso} = 4$ and the large-friction case where $Z_{iso} = 3$.

Although the value of Z is crucial for determining whether a static state is jammed or not, it is conventional to consider the states of a system in terms of the packing fraction, ϕ and the shear stress, τ, perhaps in part because the latter two quantities are easily measured. If ϕ is too low, it is not possible for Z to reach the isostatic value. However, unlike Z, there is no theoretical argument which determines a critical value of ϕ for jamming. In general, ϕ_J depends on such system properties as dimension, particle shape and polydispersity.

By applying large enough shear stress, it is usually possible to cause a jammed state to flow. Typically, the limiting yield stress, $\tau = \tau_Y(\phi)$, needed to cause an otherwise jammed system to flow increases with ϕ. In a state space of ϕ and τ, low density states are unjammed, high density states are jammed. But for τ's lying above $\tau_Y(\phi)$, flow occurs in response to shear. This scenario is encapsulated in a jamming diagram by Liu and Nagel [5], and substantiated through numerical simulations of frictionless systems [6–9]. Fig. 1a shows a sketch of the zero-temperature plane of the Liu–Nagel (L–N) jamming diagram. A key feature of this jamming diagram is a lowest $\phi = \phi_J$, such that a) all static states with $\phi < \phi_J$ are unjammed and hence stress-free, b) ϕ_J is the terminus of the yield stress curve; c) the yield stress vanishes at ϕ_J.