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Fractal analysis of the effect of particle aggregation distribution on thermal conductivity of nanofluids

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ABSTRACT

A theoretical effective thermal conductivity model for nanofluids is derived based on fractal distribution characteristics of nanoparticle aggregation. Considering two different mechanisms of heat conduction including particle aggregation and convention, the model is expressed as a function of the fractal dimension and concentration. In the model, the change of fractal dimension is related to the variation of aggregation shape. The theoretical computations of the developed model provide a good agreement with the experimental results, which may serve as an effective approach for quantitatively estimating the effective thermal conductivity of nanofluids.

1. Introduction

Quantitative estimate of the effective thermal conductivity has attracted substantial attentions since it is one of the most important parameters characterizing the heat transport properties of nanofluids [1–4]. Nanofluids are liquid suspensions that contain nanometer-size particles, with size much smaller than 100 nm, and their thermal conductivity is higher than that of their base liquids [5–8]. In recent years, a great amount of efforts has been exerted to study conductivity characteristic, and significant progress has been made towards the theoretical modeling [9–14] and laboratory experiments [15–19]. In 19th century, Maxwell [20] predicted that the thermal conductivity of mixtures increases by suspending some higher-conductivity substance such as solid particles. Since Maxwell model is only a first-order approximation, it applies only to mixtures with low particle volume fraction and small values of the ratio of thermal conductivity between particle and liquid [21]. Moreover, other traditional models for multiphase systems, such as Wiener approximation [22] and Bruggeman approach [23], fail to illuminate the abnormal enhancement of the effective thermal conductivity for low particle volume fraction in nanofluids.

Several researchers concluded that the major factors of heat conduction mechanisms in nanofluids including particle aggregation [24,25], particle motion [26–28] and liquid-layering [9,29]. Particularly, the fact that particle aggregation can enhance the effective thermal conductivity of nanofluids has been confirmed experimentally [30–32]. Wang et al. [33] claimed that particle clustering could prominently affect the enhancement of thermal conductivity of nanofluids. Hamilton and Crosser [34] presented a mixture model to explain heterogeneous two-component systems. In their model, the particle aggregation shape is invariable, which ignores the effect of aggregation shape on the effective thermal conductivity of nanofluids.

After fractal geometry was introduced by Mandelbrot [35], it became a powerful tool for the analysis of physico-geometrical properties and processes, such as electricity conductivity [36,37], spontaneous capillary imbibition [38,39], thermal conductivity [40–44] and permeability [45–48]. Several researchers [33,49–53] also apply fractal geometry to study heat conduction of nanofluids. Wang et al. [33] established an effective thermal conductivity model based on the effective medium approximation and the fractal theory to describe nanoparticle cluster and radial distribution. Xu et al. [50] applied fractal geometry to predict the thermal conductivity in terms of particles sizes distribution and heat convection of nanofluids. Considering the effect of Brownian motion of nanoparticles, Xiao et al. [52] presented a fractal model of thermal conductivity which is expressed as a function of the average diameter of nanoparticles, the nanoparticle concentration, the fractal dimension of nanoparticles and physical properties of fluids.

To the best of our knowledge, there is no full relationship to depict the effective thermal conductivity of nanofluids with fractal

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clustering distribution in terms of particle aggregation and convection. In the present study, based on Hamilton and Crosser model and Xu et al. model [34,50], an analytical model considering fractal distribution characteristic of nanoparticle aggregation is derived to estimate the effective thermal conductivity of nanofluids. The validity of the model was confirmed by comparison with the experimental results.

2. The fractal thermal conductivity model

2.1. Consideration of size effect of nanoparticles aggregation

Hamilton and Crosser [34] used empirical shape factor F to consider the effect of two heterogeneous phases and improved Maxwell equation [20] to calculate the effective thermal conductivity of nanofluid k_s that is induced by stationary nanoparticles in the liquids:

$$k_s = k_f \frac{a + (F-1) - (F-1)(1-\alpha)\phi}{a + (F-1) + (1-\alpha)\phi} \tag{1}$$

and

$$F = \frac{3}{\psi} \tag{2}$$

where $a = k_p/k_f$ (k_p is thermal conductivity of particle and k_f is thermal conductivity of fluid), ϕ is particle concentration, ψ is defined as the ratio of the surface area A'_p of a sphere to the surface area A_p of the particle whose volume V_p equal to that of the sphere, therefore

$$\psi = \frac{A'_p}{A_p} = \frac{6}{\lambda}\frac{V_p}{A_p} \tag{3}$$

where λ is aggregation size.

However, λ usually has different diameters due to aggregation in nanofluids and thus ψ is not a constant. According to Hamilton and Crosser, $\psi = 1$ for spherical particle and $\psi = 0.5$ for elliptic particle. If substituting λ, V_p and A_p with average particle size $\bar{\lambda}$, average volume $\bar{V}_p$ and average area $\bar{A}_p$, respectively, Eq. (3) can be deduced as

$$\psi = \frac{A'_p}{A_p} = \frac{6}{\bar{\lambda}}\frac{\bar{V}_p}{\bar{A}_p} \tag{4}$$

It has been shown that the size distribution of aggregation in nanofluids follows the fractal power law [33,49,50]. Analogous to pores in fractal porous media, the fractal probability density function can be expressed as [50]

$$f(x) = D\lambda_{\min}^{D}\lambda^{-(D+1)} \tag{5}$$

The fractal dimension D is determined by [48]

$$\xi = \phi^{\frac{1}{D_E - D}} \quad \text{or} \quad D = D_E - \frac{\ln\phi}{\ln\xi} \tag{6}$$

where $D_E = 3$ for three-dimension space and $\xi = \lambda_{\min}/\lambda_{\max}$, where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum diameters of nanoparticle cluster, respectively. When the particle cluster has fractal characteristics, its area and volume are $\pi\lambda^2$ and $\pi/6 \cdot \lambda^3$, respectively, Eq. (4) can be expressed

$$\psi = \frac{6}{\bar{\lambda}} \frac{\int_{\lambda_{\min}}^{\lambda_{\max}} \frac{\pi}{6}\lambda^3 f(\lambda)d\lambda}{\int_{\lambda_{\min}}^{\lambda_{\max}} \pi\lambda^2 f(\lambda)d\lambda} \tag{7}$$

Combining Eqs. (5), (6) and (7), ψ can be obtained as

$$\psi = \frac{2-D}{3-D}\frac{\lambda_{\min}}{\bar{\lambda}}\frac{\phi^{-1}-1}{\phi^{\zeta_2}-1} \tag{8}$$

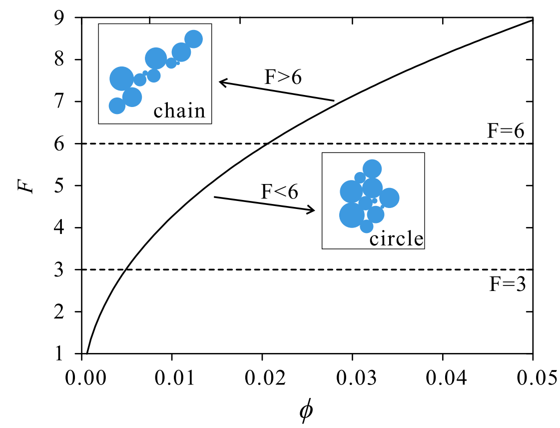

Fig. 1. Relationship between F and concentration ϕ in Eq. (11). The dashed line for $F = 3$ and $F = 6$ [34] representing respectively sphere and ellipse for suspended aggregation.

where $\zeta_2 = (D-2)/(3-D)$ and $\bar{\lambda}$ can be found from the statistical property of fractal object [50], as

$$\bar{\lambda} \approx \frac{D}{D-1}\lambda_{\min} \tag{9}$$

Inserting Eq. (9) into Eq. (8), the following equation can be obtained

$$\psi = \frac{D-1}{D}\frac{2-D}{3-D}\frac{\phi^{-1}-1}{\phi^{\zeta_2}-1} \tag{10}$$

Therefore, inserting Eq. (10) into Eq. (2) yields

$$F = 3\frac{D}{D-1}\frac{3-D}{2-D}\frac{\phi^{\zeta_2}-1}{\phi^{-1}-1} \tag{11}$$

In Hamilton and Crosser's model, F is constant for same shape particles ($F = 6$ for ellipse and $F = 3$ for sphere). However, it is observed that F is the function of fractal dimension and concentration as expressed in Eq. (11), and F increases with the increasing of concentration (see Fig. 1). As shown in Fig. 1, considering fractal distribution of nanoparticle aggregation, the shape of aggregation gradually grows to chain with the increasing concentration. When $F < 6$, most aggregation shapes are circles. Combination of Eqs. (1) and (11) is the presented fractal model that predicts effective thermal conductivity of nanofluids relating with the effect of nanoparticles cluster distribution.

2.2. Consideration of convention effect of nanoparticles aggregation

Heat convection due to the Brownian motion of nanoparticles could enhance heat transfer in nanofluids. While most convention models are based on an assumption that suspended aggregation in nanofluids have uniform diameter. Xu et al. [50] theoretically analyzed thermal conductivity k_c for heat convection by using the fractal geometry for different sizes of nanoparticle cluster, which can be expressed as

$$k_c = c\frac{k_f \cdot Nu \cdot d_f}{\Pr}\frac{D(2-D)}{(1-D)^2}\frac{(\xi^{1-D}-1)^2}{\xi^{2-D}-1}\frac{1}{\bar{\lambda}} \tag{12}$$

where c is an empirical constant, Nu is the Nusselt number for liquid flowing around a sphere, Pr is the Prandtl number for fluids and d_f is diameter of liquid molecule. Combining Eqs. (6) and (12), the following equation can be obtained

$$k_c = c\frac{k_f \cdot Nu \cdot d_f}{\Pr}\frac{(2-D)D}{(1-D)^2}\frac{(\phi^{\zeta_1}-1)^2}{\phi^{\zeta_2}-1}\frac{1}{\bar{\lambda}} \tag{13}$$