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Comment

Comment on “Compact envelope dark solitary wave in a discrete nonlinear electrical transmission line” [Phys. Lett. A 373 (2009) 3801–3809]

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ABSTRACT

We revisit the derivation of the equation modeling envelope waves in a discrete nonlinear electrical transmission line (NLTL) considered a few years back in Physics Letters A 373 (2009) 3801–3809. Using a combination of rotating wave approximation and the Gardner–Morikawa transformation, we show that the modulated waves are described by a new type of extended nonlinear Schrödinger equation. In addition the expressions of several coefficients of this equation are found to be strongly different from those given earlier. As a consequence, key relationships between these coefficients that sustained the previous analysis are broken.

In Ref. [1], a discrete nonlinear electrical transmission line (NLTL) governed by the equation

$$\begin{aligned}\frac{\mathrm{d}^2 V_n}{\mathrm{d}t^2} + u_0^2\left(2V_n - V_{n+1} - V_{n-1}\right) + \omega_0^2 V_n - \alpha\frac{\mathrm{d}^2 V_n^2}{\mathrm{d}t^2} + \beta\frac{\mathrm{d}^2 V_n^3}{\mathrm{d}t^2} = C_{0r}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Big\{(V_{n-1}+V_{n+1}-2V_n)\\ + \eta\left[(V_{n-1}-V_n)^2 - (V_n - V_{n+1})^2\right] + \lambda\left[(V_{n-1}-V_n)^3 - (V_n-V_{n+1})^3\right]\Big\}\end{aligned} \tag{1}$$

was considered with the aim of analyzing the propagation of modulated waves in it. To this end, the authors adopted the rotating wave approximation (RWA), according to which the time dependence of the voltage at lattice site n is expressed approximately as

$$V_n(t) = A(\xi,\tau)e^{i\theta} + A^*(\xi,\tau)e^{-i\theta}, \tag{2}$$

where the asterisk denotes complex conjugation and the function $A(\xi,\tau)$ is to be determined. The phase variable θ and the envelop variables ξ and τ are defined as

$$\theta = kn - \omega t, \quad \xi = \varepsilon\,(n - \mu t), \quad \tau = \varepsilon^2 t; \tag{3}$$

where the angular frequency ω and the wavenumber k are related by the linear dispersion relation

$$\omega^2 = \frac{\omega_0^2 + 4u_0^2\sin^2\left(\frac{k}{2}\right)}{1 + 4C_{0r}\sin^2\left(\frac{k}{2}\right)}, \tag{4}$$

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and

$$\mu = \frac{1}{\omega} \frac{\left(u_0^2 - C_{0r}\omega^2\right) \sin k}{1 + 4C_{0r} \sin^2\left(\frac{k}{2}\right)} \tag{5}$$

is the linear group velocity. It is obvious from Eq. (2) that the voltage at any site shifted by an integer m from a reference site n is given by

$$V_{n+m}(t) = [A(\xi + \varepsilon m, \tau) e^{ikm}] e^{i\theta} + [A^*(\xi + \varepsilon m, \tau) e^{-ikm}] e^{-i\theta}. \tag{6}$$

Assuming that the dimensionless parameter ε is small ($0 < \varepsilon \ll 1$), Taylor expansions of the following form are warranted:

$$A(\xi + \varepsilon m, \tau) = A(\xi, \tau) + \varepsilon m \frac{\partial A}{\partial \xi}(\xi, \tau) + \frac{\varepsilon^2 m^2}{2} \frac{\partial^2 A}{\partial \xi^2}(\xi, \tau) + \frac{\varepsilon^3 m^3}{6} \frac{\partial^3 A}{\partial \xi^3}(\xi, \tau) + \cdots \tag{7}$$

Here the specific values $m = 1$ and $m = -1$ are relevant for Eq. (1). The use of the variables θ, ξ and τ defined in Eq. (3) also implies that the time derivative operators are transformed according to

$$\frac{\mathrm{d}}{\mathrm{d}t} = -\omega \frac{\partial}{\partial \theta} - \varepsilon \mu \frac{\partial}{\partial \xi} + \varepsilon^2 \frac{\partial}{\partial \tau}, \tag{8a}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} = \omega^2 \frac{\partial^2}{\partial \theta^2} + \varepsilon\,(2\omega\mu) \frac{\partial^2}{\partial \theta \partial \xi} + \varepsilon^2 \left[\mu^2 \frac{\partial^2}{\partial \xi^2} - 2\omega \frac{\partial^2}{\partial \theta \partial \tau}\right] - 2\mu\varepsilon^3 \frac{\partial^2}{\partial \xi \partial \tau} + \varepsilon^4 \frac{\partial^2}{\partial \tau^2}. \tag{8b}$$

The scaling of the slow variables ξ and τ through the introduction of the dimensionless parameter ε was not explicitly considered in [1]; yet it is a standard one [2–5] which makes the calculations convenient. In effect, the authors explained that upon substituting the ansatz (2) in the lattice equation (1), they neglected second order derivatives with respect to τ and those of order higher than two with respect to ξ. By simply expecting Eq. (7) and Eq. (8b), it should be clear that this can be *consistently* accomplished by neglecting all terms of order higher than two in the power series expansion in ε at the last stage of the required calculations.

Now, using $c.c.$ as a short hand for "complex conjugate of the preceding terms" and as an example of these computations, we have

$$V_n^3 = 3|A|^2 A e^{i\theta} + A^3 e^{3i\theta} + c.c. \tag{9}$$

and

$$\begin{aligned} \frac{\mathrm{d}^2 V_n^3}{\mathrm{d}t^2} &= \left[-3\omega^2 |A|^2 A + 6i\varepsilon\mu\omega \frac{\partial\left(|A|^2 A\right)}{\partial \xi} + \varepsilon^2 \left(3\mu^2 \frac{\partial^2\left(|A|^2 A\right)}{\partial \xi^2} - 2i\omega \frac{\partial\left(|A|^2 A\right)}{\partial \tau}\right) + \mathcal{O}(\varepsilon^2)\right] e^{i\theta} \\ &+ \left[-9\omega^2 A^3 + 6i\varepsilon\mu\omega \frac{\partial\left(A^3\right)}{\partial \xi} + \varepsilon^2 \left(\mu^2 \frac{\partial^2\left(A^3\right)}{\partial \xi^2} - 6i\omega \frac{\partial\left(A^3\right)}{\partial \tau}\right) + \mathcal{O}(\varepsilon^2)\right] e^{3i\theta} + c.c. \end{aligned} \tag{10}$$

In the RWA only the coefficient of $e^{i\theta}$ is considered in the establishment of the envelop equation. We can note that the contribution of $\mathrm{d}^2 V_n^3/\mathrm{d}t^2$ to this envelop equation for the line under consideration does not include only the nonlinear cubic term $|A|^2 A$, but also terms involving first order derivatives with respect to τ and others with first and second order derivatives with respect to ξ. In other words, as per the last term on the left hand side of Eq. (1), the equation describing envelop waves in the system should contain terms whose coefficients depend on β and which involve derivatives either with respect to τ or ξ. This is clearly not the case of Eq. (8) in [1]. In fact, after careful and lengthy but otherwise straightforward calculations we find that, within the RWA, the dynamics of the envelop $A(\xi, \tau)$ is governed by the following nonlinear partial differential

$$\begin{aligned} i\frac{\partial A}{\partial \tau} + P\frac{\partial^2 A}{\partial \xi^2} + Q\,|A|^2 A &= ir_1 \frac{\partial A}{\partial \xi}|A|^2 + r_2 \frac{\partial^2 A}{\partial \xi^2}|A|^2 + r_3 A^2 \frac{\partial^2 A^*}{\partial \xi^2} + r_3' A \left|\left(\frac{\partial A}{\partial \xi}\right)\right|^2 \\ &+ r_4 A^* \left(\frac{\partial A}{\partial \xi}\right)^2 + i\sigma \left[A^2 \frac{\partial A^*}{\partial \tau} + 2\frac{\partial A}{\partial \tau}|A|^2\right] - i\frac{\sigma\mu}{\varepsilon} A^2 \frac{\partial A^*}{\partial \xi}. \end{aligned} \tag{11}$$

The above relation is a modified extended nonlinear Schrödinger (MENLS) equation which comprises classical terms in its left hand side while non-standard terms are gathered in its right hand side. The expressions of the various coefficients of this equation are given respectively as follows:

$$P = \frac{1}{2\omega} \frac{u_0^2 \cos(k) - \mu^2 - \left[4\omega\mu \sin(k) + 4\mu^2 \sin^2\left(\frac{k}{2}\right) + \omega^2 \cos(k)\right] C_{0r}}{1 + 4C_{0r} \sin^2\left(\frac{k}{2}\right)}, \tag{12a}$$

$$Q = \frac{3\omega}{2\varepsilon^2} \frac{\beta + 16 C_{0r} \lambda \sin^4\left(\frac{k}{2}\right)}{1 + 4C_{0r} \sin^2\left(\frac{k}{2}\right)}, \tag{12b}$$

$$r_1 = \frac{6}{\varepsilon} \frac{\mu\beta + 8\sin^3\left(\frac{k}{2}\right)\left[\omega \cos\left(\frac{k}{2}\right) + 2\mu \sin\left(\frac{k}{2}\right)\right] C_{0r}\lambda}{1 + 4C_{0r} \sin^2\left(\frac{k}{2}\right)}, \tag{12c}$$