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Emergence of amplitude death scenario in a network of oscillators under repulsive delay interaction

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A B S T R A C T

We report the existence of amplitude death in a network of identical oscillators under repulsive mean coupling. Amplitude death appears in a globally coupled network of identical oscillators with instantaneous repulsive mean coupling only when the number of oscillators is more than two. We further investigate that, amplitude death may emerge even in two coupled oscillators as well as network of oscillators if we introduce delay time in the repulsive mean coupling. We have analytically derived the region of amplitude death island and find out how strength of delay controls the death regime in two coupled or a large network of coupled oscillators. We have verified our results on network of delayed Mackey–Glass systems where parameters are set in hyperchaotic regime. We have also tested our coupling approach in two paradigmatic limit cycle oscillators: Stuart–Landau and Van der Pol oscillators.

1. Introduction

Oscillation quenching is an important, fundamental and emergent phenomena for coupled oscillatory systems [1,2]. Suppressing oscillations of coupled systems and achieving desired stable states are the key features of neuronal interaction [3], lasers [4], cellular differentiation [5], even in optical experiments [6]. These stable states are distinctly classified into two categories: Amplitude death (AD) where all the oscillators collapse to single fixed point and oscillation death (OD) where all the oscillators stabilize to new multiple fixed points which are created due to interactions [1,7]. To stabilize the fixed points (creation of AD or OD) in coupled oscillatory systems, two control strategies are highly used in respect to nonlinear research. One is to introduce parameter mismatch in diffusively coupled oscillators [8,9] and the other is sufficient time delayed interaction in coupled identical oscillators [10]. In last decade, other strategies are also implemented for suppression of oscillations: conjugate coupling [11], environment coupling[12], dynamic coupling [13], mean-field coupling [14], nonlinear coupling [15] and an effect of additional repulsive link [7] and mixed type coupling [16] in diffusively coupled oscillators. Amplitude death scenario is also reported under partially time-delayed coupling [17] and mixed time-delayed coupling [18].

In the oscillation quenching state, each dynamical oscillatory unit of a coupled network loses it oscillation by reaching stable fixed points (AD or OD) due to the coupling interaction. This oscillation quenching state has many potential applications in real systems where the oscillation is an unwanted situation. This is important to suppress the unwanted oscillations in laser systems [2], synthetic genetic oscillator [19], cellular differentiation [5] etc.

We emphasize here that, most of the stabilization techniques introduced for cessation of oscillation (using delayed or non-delayed interaction) have been devoted to attractively coupled dynamical elements since the entrainment between oscillators is one of the main concerns. However, in some cases, the coupling can be considered as repulsive when negative sign in the coupling strength repels each other resulting in out-phase behavior. Anti-phase synchronization is observed with strong repulsive interaction [20] and also verified experimentally in the electrically coupled biological neurons [21]. For example, it is well known that biological networks are connected attractively (which is considered to be related to excitatory synapses) and repulsively (which is considered to be related to inhibitory synapses) to improve synchronization and transmission performance in the brain [22]. Firing pattern including multistability and chaotic firing is observed in coupled excitable neurons when they are coupled repulsively [23]. Another important example of repulsive coupling in biological sciences is in the interaction between the ion channels of a cell membrane containing voltage dependent ion channels [24]. In a population of synthetic genetic oscillators, they are suppressed by phase repulsive coupling [25]. On the other hand solitary state

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emerges in networks of globally coupled identical oscillators with repulsive and attractive coupling [26]. Recently, Hens et al. [27] have observed the death scenario (transition from amplitude to oscillation death) in diffusively coupled identical oscillators under additional repulsive links. Remarkably, this type of transition is also verified and tested in a case of a novel cyclic interaction when an asymmetry is introduced in terms on negative parameter mismatch [28]. So far, to the best of our knowledge, in all the existing works on amplitude death are based on attractive instantaneous or time-delayed diffusive coupling and no one has checked how the oscillatory dynamics of a network can be controlled or suppressed when links are repulsively connected only.

In this letter, we concentrate on death scenario between network of identical oscillators using repulsive mean coupling. Here the repulsive mean coupling is expressed by the average of two neighboring state as $k(x+y)$ between the two coupled systems $\dot{x}=f(x)$ and $\dot{y}=f(y)$; $x\in R^n$, $y\in R^n$, where the coupling strength k is negative sign. Anti-phase synchronization emerges when two identical oscillators are interacted via repulsive mean coupling [29]. To testify the effect of repulsive mean interaction, we have chosen a delayed Mackey Glass oscillator [30] as a chaotic model and Landau–Stuart as well as Van der Pol (VDP) oscillator [8,10,31] as limit cycle models. Both the systems have trivial fixed points at origin. Our objective is to stabilize these trivial fixed points i.e. creating amplitude death in network where the links are repulsively connected only. We identify that instantaneous repulsive mean coupling induces amplitude death in a network of identical oscillators when the number of oscillators is more than two. For two identical oscillators, amplitude death may be generated if we introduce a coupling delay in the repulsive mean coupling. More surprisingly, we check that a mixture of diffusive and repulsive delay interaction may control (increase or decrease) the death island for two coupled oscillators.

The plan of the letter is as follows: In next Sec. 2, we discuss amplitude death scenario in network of identical Mackey–Glass systems [30] where the parameters of the systems are chosen in hyperchaotic regime. For simplicity we first observe that the time-delay or non-time delayed repulsive mean coupling can induce amplitude death in three coupled oscillators for a certain coupling strength where as, in two coupled oscillators, a delay coupled repulsive interaction is necessary to create AD. Next we will discuss that, if we introduce a time-delay interaction in the repulsive mean coupling, reviving of oscillation from AD state is also a possible scenario after a certain critical coupling. We further show that, delay or non-delay repulsive interaction may induce AD in a network of coupled Mackey–Glass oscillators of size 100. Next we check, how delayed or non-delayed repulsive interaction affects in the network of limit cycle oscillators. As a paradigmatic model, we have chosen coupled Stuart–Landau oscillators in Sec. 3. In that network of size N, the characteristic equation for the stabilization of fixed points has been derived analytically. At first, to track the proper death island, our analytical prediction has been validated with two Stuart–Landau oscillators when they are coupled via delayed repulsive interaction is symmetrically or asymmetrically. We find out the amplitude death region in (τ,k) plane for different values of asymmetric parameter p. We have checked that size of the death region is controlled by the asymmetric (p) delayed effect. Further we validate our coupling scheme and analytical result in a network of 100 Stuart–Landau oscillators. Next the effect of delayed/non-delayed repulsive interaction is also verified in another network of limit cycle oscillators: Van der Pol oscillator in Sec. 4. The analytical prediction of death island in a small network of size three has been checked in the absence of delay coupling. Further a τ–k space is drawn to isolate the death island in a large network (size 100) of Van der Pol oscillator. In the Sec. 5, effect of mixed delay repulsive mean coupling is discussed using Stuart–Landau oscillator. Finally, we summarize our results in Sec. 6.

2. Network of coupled identical chaotic oscillators under repulsive mean coupling

We construct a general framework of globally coupled network of oscillators under repulsive delay coupling as

$$\dot{X}_j = F(X_j) - k\sum_{m=1,m\neq j}^{N}[X_m(t-\tau_m)+X_j], \quad j=1,2,\ldots,N \quad (1)$$

where $\dot{X}_j=F(X_j)$ governs the local dynamics of the vector field X_j in each node, $k(>0)$ is the repulsive coupling strength, N is the number of oscillators in the network, and $\tau_1,\tau_2,\ldots,\tau_N$ are the delay times in the repulsive mean coupling.

First, we examine whether our proposition of repulsive delay interaction scheme may work on coupled chaotic oscillators. To elaborate the effect of the coupling scheme we use coupled Mackey–Glass oscillators [30] (as chaotic model). The network of oscillators under delayed repulsive mean coupling is written as

$$\dot{x}_j = -ax_j + \frac{bx_j(t-\tau)}{1+x_j^{10}(t-\tau)} - k\sum_{m=1,m\neq j}^{N}[x_m(t-\tau_m)+x_j] \quad (2)$$

where $j=1,2,\ldots,N$. For simplicity, we consider all the repulsive coupling delays are identical i.e. $\tau_1=\tau_2=\cdots=\tau_N=\tau_c$. We choose the parameters as $a=0.1$, $b=0.2$, $\tau=32.0$ so that the individual node oscillates hyper chaotically in absence of coupling strength. The chaotic time series and attractor in $(x(t)$ vs. $x(t-\tau))$ plane are shown in Fig. 1(a). To understand the effect of delayed repulsive interaction we start with two coupled Mackey–Glass oscillators $(N=2)$ when the coupling interaction is completely instantaneous i.e. $\tau_1=\tau_2=0.0$. Such instantaneous interaction fails to suppress the oscillatory states of each unit rather generates anti-synchronization after a critical value of coupling strength k. For $k=0.2$, the time series of each $(x_1(t),x_2(t))$ is shown in Fig. 1(b) by blue and red color respectively. The anti-synchronization error $x_1(t)+x_2(t)$ is also shown by black color which follows a zero line. The observed result concludes that non-delay repulsive interaction cannot stabilize the origin of two coupled Mackey–Glass chaotic units. But two repulsively coupled identical Mackey–Glass systems produce amplitude death in presence of coupling delay time (results are not shown).

Next, we consider $N=3$, i.e. three identical Mackey–Glass oscillators are coupled by instantaneous/non-delay ($\tau_1=\tau_2=\tau_3=\tau_c$; $\tau_c=0.0$) mean repulsive interaction. The individual Mackey–Glass system has one of the equilibrium point at origin. Extrema of the variable of one node has been plotted in Fig. 1(c) as a function of k and it is clear that amplitude death occurs after $k\geq 0.1$ via inverse Hopf bifurcation and the fixed point (origin) remain stable for higher coupling strength. Next we check the delay effect in the repulsively coupled three Mackey–Glass nodes. Contrast to the previous result, when coupling delay $\tau_c=7.0$ is introduced in the coupling term, we observe (Fig. 1(d)) that amplitude death occurs in $0.1\leq k<0.21$ and surprisingly find that there is a critical value of coupling $k=0.21$ above which the stable equilibrium point destabilizes and oscillatory state revives. This phenomenon emerges due to the presence of time delay τ_c in the repulsive mean coupling. Keeping fixed the coupling strength at $k=0.35$ we have changed τ_c and find the bifurcation diagram of the variable of one node and a transition from AD to oscillation occurs (Fig. 1(e)). The stability of origin changes if the complex eigenvalue λ of the characteristic equation crosses the imaginary axis $\lambda=i\beta$ and regaining of oscillation via Hopf bifurcation due to the