Physics Letters A 374 (2010) 2017–2023

Contents lists available at ScienceDirect

Physics Letters A

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Modified function projective synchronization of hyperchaotic systems through Open-Plus-Closed-Loop coupling

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ARTICLE INFO

Article history:
Received 3 December 2009
Received in revised form 10 February 2010
Accepted 25 February 2010
Available online 3 March 2010
Communicated by A.R. Bishop

Keywords:
Hyperchaotic Rossler system
Hyperchaotic Lu system
Hyperchaotic systems
Chaos synchronization
Modified function projective synchronization (MFPS)
Open-Plus-Closed-Loop (OPCL) coupling

ABSTRACT

Recently introduced modified function projective synchronization (MFPS) in which chaotic systems synchronize up to a scaling function matrix has important applications in secure communications. We design coupling function for unidirectional coupling in identical and mismatched hyperchaotic oscillators to realize MFPS through Open-Plus-Closed-Loop (OPCL) coupling method. The arbitrary scaling function matrix elements are properly chosen such that we can produce function projective synchronization, synchronization, anti-synchronization and amplitude death on desired states of the response system simultaneously. Numerical simulations on identical hyperchaotic Rossler and mismatched hyperchaotic Lu system are presented to verify the effectiveness of the proposed scheme. A secure communication scheme based on MFPS is also presented.

1. Introduction

Chaos synchronization is an important topic both theoretically and practically and has been studied in the last three decades because of its potential applications in secure communications [1], nano oscillators [2], biological systems [3]. Since Pecora and Carrol [4] introduced a method to synchronize two identical systems with different initial conditions, a variety of approaches have been proposed for the synchronization of chaotic systems which include complete synchronization [4], phase synchronization [5], generalized synchronization [6], lag synchronization [7], intermittent lag synchronization [8], time scale synchronization [9], intermittent generalized synchronization [10], projective synchronization [11], modified projective synchronization [12,13] and function projective synchronization [14,15]. Apart from synchronization, realizing amplitude death [16] in the response system is also an interesting task.

Recently H. Du et al. [17] studied a more general form of function projective synchronization called modified function projective synchronization (MFPS) where the drive and response systems are synchronized up to a desired scaling function matrix in Lorenz system by active control method. The unpredictability of scaling functions in MFPS can provide additional security in secure communication. K.S. Sudheer et al. [18] reported MFPS through adaptive control method.

A more physically realizable Open-Plus-Closed-Loop (OPCL) coupling method, introduced by I. Grosu et al. [19,20] has recently been used to obtain projective synchronization of two identical and mismatched chaotic oscillators. To our knowledge, there are no reports of FPS through OPCL coupling in chaotic systems. Moreover, hyperchaotic systems possessing at least two positive Lyapunov exponents have more complex behaviour and abundant dynamics than chaotic systems and are more suitable for some engineering applications such as secure communication. Hence how to realize synchronization of hyperchaotic systems is an interesting and challenging work.

Motivated by the above discussions, in this Letter we propose a scheme for realizing the MFPS of hyperchaotic systems through OPCL coupling. We design controllers to obtain MFPS of two identical hyperchaotic Rossler systems and mismatched hyperchaotic Lu systems. A secure communication scheme based on MFPS is also investigated. The organization of the rest of this Letter is as follows. Section 2 briefly describes MFPS through OPCL coupling. In Section 3 we design controllers for MFPS of two identical Rossler hyperchaotic systems. Section 4 presents MFPS of two mismatched Lu hyperchaotic systems. A secure communication scheme based on MFPS of Lu hyperchaotic system is presented in Section 5. Finally some concluding remarks are given in Section 6.

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2010.02.068

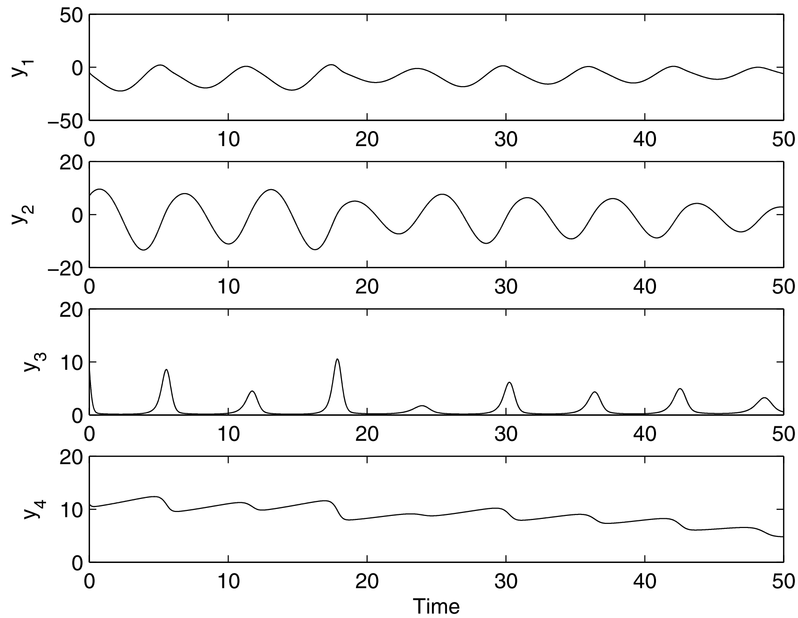

Fig. 1. Time evolution of the drive system (8) states.

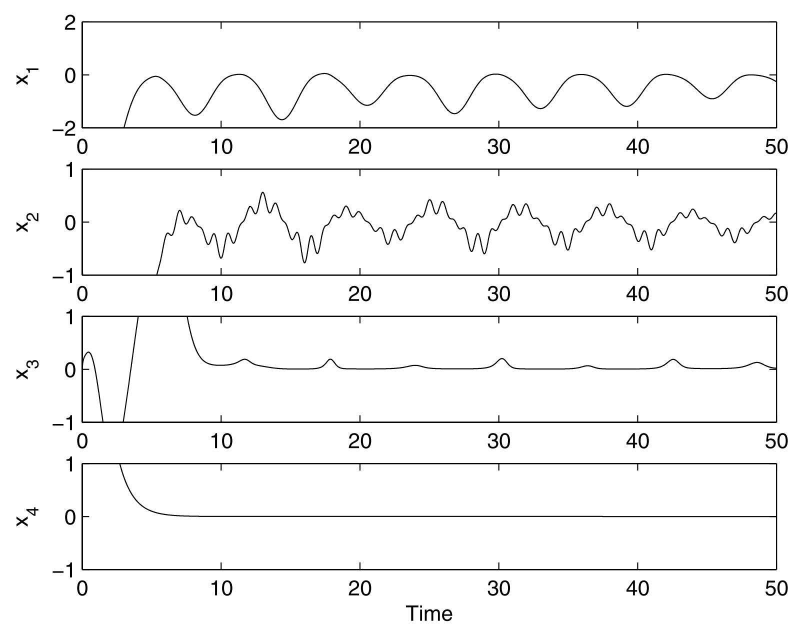

Fig. 2. Time evolution of the response system (9) states.

2. MFPS through OPCL coupling

We first briefly discuss the OPCL method for the FPS of two mismatched oscillators. A chaotic driver is defined by

$$\dot{y} = f(y) + \Delta f(y), \quad y \in R^n, \tag{1}$$

where $\Delta f(y)$ contains the mismatched terms which is zero for identical oscillators. To realize FPS, it drives another chaotic oscillator $\dot{x} = f(x)$, $x \in R^n$ to achieve a goal dynamics $g(t) = m(t)y(t)$, where $m(t)$ is an arbitrary scaling function. After coupling, the response system is given by

$$\dot{x} = f(x) + D(x, g), \tag{2}$$

where the coupling function is defined as

$$D(x, g) = \dot{g} - f(g) + \left(H - \frac{\partial f(g)}{\partial g}\right)(x - g), \tag{3}$$

$\frac{\partial f(g)}{\partial g}$ is the Jacobian of the dynamical system and H is an arbitrary constant Hurwitz matrix $(n \times n)$ whose eigen values all have negative real parts. The error signal of the coupled system is defined by $e = x - g$ and $f(x)$ can be written using Taylor series expansion, as

$$f(x) = f(g) + \frac{\partial f(g)}{\partial g}(x - g) + \cdots. \tag{4}$$

Keeping the first-order terms in Eq. (3) and substituting in Eq. (2), the error dynamics is obtained as

$$\dot{e} = He. \tag{5}$$

Since H is a Hurwitz matrix which has all its eigen values have negative real parts, $e \to 0$ as $t \to \infty$ and we obtain asymptotic FPS.