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Current and future prospects for the application of systematic theoretical methods to the study of problems in physical oceanography

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ABSTRACT

This essay is a commentary on the pivotal role of systematic theoretical methods in physical oceanography. At some level, there will always be a conflict between theory and experiment/data collection: Which is pre-eminent? Which should come first? This issue appears to be particularly marked in physical oceanography, to the extreme detriment of the development of the subject. It is our contention that the classical theory of fluids, coupled with methods from the theory of differential equations, can play a significant role in carrying the subject, and our understanding, forward. We outline the philosophy behind a systematic theoretical approach, highlighting some aspects of equatorial ocean dynamics where these methods have already been successful, paving the way for much more in the future and leading, we expect, to the better understanding of this and many other types of ocean flow. We believe that the ideas described here promise to reveal a rich and beautiful dynamical structure.

1. Introduction

By-and-large, modern oceanography is a field of data-intensive science. There is no denying that some data is needed to support any new idea or some – apparently amazing – insight. It is quite evident that the interpretation of available data often leads to important correlations, suggests hidden patterns, or exposes unexpected phenomena – for example, the Equatorial Undercurrent (EUC), one of the major oceanic currents, was discovered in 1952 when Townsend Cromwell examined the field data for the subsurface ocean flow in the neighbourhood of the Pacific Equator. The collection of enough relevant data will necessarily lead to correlations between various factors, but the classical adage 'correlation does not imply causation' must remain at the forefront whenever data-driven knowledge is the dominant way of thinking. Throughout the 19th and the early part of the 20th centuries, science coped with nature's complexities by seeking the underlying simplicities hidden in the sparse data acquired by observation and experiment at the time. Over this period, it is reasonable to argue that progress occurred by subordinating experiment to theory. However, nowadays, the availability of massive datasets reinforces the approach that urges us to identify patterns by using (mainly) statistical methods to detect significant relationships, and numerical simulations to test their veracity and relevance. Who could argue with this?

Well, we do not believe that 'big data' should reign supreme: we wish to express the view that, in physical oceanography, a data-driven approach does not, and cannot, answer the fundamental questions about these complex systems. In our opinion, the pendulum has swung too far into the data-collection and interpretation camp, apparently leaving the theoreticians in the wilderness. While theories without data are mere speculation – certainly not credible science – data (even lots of data), unaccompanied by an explanatory theory, is virtually useless. The danger here is the increased risk of finding spurious correlations – statistically robust but unimportant associations – or missing altogether the underlying structure. And of course, the more data, the more the likelihood that superficial patterns will vastly outnumber genuine discoveries! Furthermore, the advent of high-performance computers has had a major impact on modern studies of natural phenomena. Certainly, the outcomes of computational fluid dynamics are enjoyable, often instructive and, in some instances, they can provide insight into issues that could not be addressed otherwise. However, without a theoretical understanding of the underlying physical processes, computer simulations, just like experiments or field observations, can be just a vast catalogue of intriguing numbers and pictures. Even an apparently successful simulation of real-world phenomena

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can hide important mechanisms behind a plethora of detail needed to produce the results; these may obscure, rather than provide, the hoped-for insights. Thinking that theory is obsolete (since we are now able to measure everything and thus turn every question into no more than a numerical exercise), is an illusion. Indeed, we believe that this approach is tantamount to putting the cart before the horse: some data might suggest a way forward, but the useful interpretation of data requires a theory. Let us be clear: by theory we mean a fundamental examination of the problem – not some *ad hoc* modelling – based on a systematic analysis, being careful about the formulation, overarching assumptions, approximations, and so on. Such an approach will give us confidence in any conclusions that are reached. The history of physical discoveries suggests that the quest for theories expressed in terms of equations, typically exhibiting inherent mathematical beauty and simplicity, is the way forward: they have usually stood the test of time by revealing fundamental, underlying processes that greatly improve our understanding and often lead to many, varied and useful applications.

In this article, we will discuss some specific topics in physical oceanography where, by means of a systematic mathematical approach, one is able to gain some fundamental understanding of the processes involved. The main thrust of our argument is that the tried-and-tested techniques of classical fluid mechanics – which seem to have been relegated to a fairly minor role in the current work in oceanography – combined with methods from partial differential equations and dynamical systems, have an important and enduring role to play. We advocate a theory-based approach to underpin both data collection and the interpretation of the results, thus preventing the dubious findings that are often inherent in solely manipulating big data (some of which are highlighted in [1]). For example, data assimilation – a versatile, modern methodology that relies on statistical techniques to combine data with numerical simulations of a model, aiming to produce a quantitative estimate of the state of a specific real-world system – is widely used [2,3]. While data assimilation undoubtedly plays an important role in ocean science, one should keep in mind that all models are, to some extent, approximate and that all data sets are incomplete; moreover, it is often not possible to disentangle observational noise and model error with any certainty. In particular, while data assimilation appears to work well for ocean regions away from the Equator, it has been found particularly inaccurate for equatorial oceanic flows: e.g. it unrealistically forecasts large vertical velocities extending to considerable depths [4]. Much attention has been paid to the issue of data calibration but, in our view, this amounts to applying *ad hoc* remedies to the symptoms alone – there is a need to improve the basic underlying theoretical framework. This is made even more compelling by certain features that are specific to equatorial flows: since the meridional component of the Coriolis force vanishes there, the Equator works as a (fictitious) natural boundary that facilitates azimuthal-flow propagation, and also leads to the widely-used mid-latitude geostrophic balance (between the Coriolis force and the meridional pressure gradient) breaking down in equatorial regions. Consequently, any data-driven approach must, we argue, always be guided by theory in its quest to discover the mechanisms that drive ocean dynamics. We will indicate how, even with the complex dynamics of the equatorial ocean, where the effects of stratification and of depth-dependent currents are important, considerable success is possible when the problem is treated as a study in theoretical fluid mechanics, of flows with a free surface over a rigid bed. One particular region that comes to mind is the oceanic flow in the neighbourhood of, and predominantly along, the Pacific Equator; here resides the Equatorial Undercurrent (EUC) and its associated thermocline. The overall structure of this flow, along its total length of over 11,000 km, is well known, but its dynamic properties, and the interactions between its various elements, are poorly understood at a fundamental level. We focus on the ocean dynamics in the equatorial Pacific as an enlightening case study for arguing that systematic theoretical approaches are essential for a genuine understanding of these physically complex systems.

2. Modelling of ocean flows

A precise and systematic mathematical approach to any problem in applied mathematics starts from a set of general, overarching, governing equations. In the context of fluid mechanics, this is either the Euler equation (for inviscid flow), or the Navier–Stokes equation (for a viscous fluid), together with a suitable equation of mass conservation. The process of isolating the particular problem of interest then involves, first, the choice of working with a viscous or inviscid fluid. For ocean flows it is fairly usual to work with the Euler equation because the most significant effects of viscosity in the open ocean are to produce wave-amplitude reduction, and diffusion of the deeper motions, over times and distances that are far larger than those of the dynamical processes of most interest [5,6]. Furthermore, the dynamic conditions at the surface, which may be thought to require some crucial viscous component, can be treated by a combination of a variable pressure on the surface and the transfer of momentum from the wind to the waves on the inclined surface of the wave. Second, we almost always require the selection of some suitable scales and associated nondimensional parameters and approximation; to illustrate the vast range of temporal and spatial scales present in the ocean, note that any complete analysis must contend with eddies that span mere centimetres and last a few seconds on the one hand, and large-scale wave-current interactions (involving waves with wavelengths of tens to hundreds of kilometres and periods of seconds to hours, and currents having spatial extents of thousands of kilometres and lifetimes measured in centuries) on the other (see [7]). Finally, of course, we need to specify an appropriate set of boundary and initial conditions. From this prescription follows an important observation: any model equation or system that cannot be interpreted in these terms is, necessarily, inconsistent with the underlying mathematical structure of the problem and then, perforce, its analysis is a wasteful exercise. So, with our general formulation in place, how might we proceed?

Clearly, we would wish to solve the complete problem, as originally posed, in all its generality, but this is not going to be possible. We must, with care, attempt some meaningful simplifications. It is at this stage that we might invoke some suitable guiding principles, and what we choose is likely to be driven by what we know about the system, our skills and what we believe is reasonably accessible. Predominantly, the plan will be to identify some important aspect of the problem that is worthy of attention, and make simplifying assumptions (always completely consistent with the original formulation) in order to bring this aspect to the fore. This approach can be used piece-meal to examine, typically, one property or another of the system, the complete solution being some mix of all these (but this mix is not accessible by any consistent mathematical technique). Although this means that we are analysing, in turn, various simpler problems, we can be sure that we have a robust basis for the results that are generated, and we can be very sure of the underlying assumptions and simplifications that produced each one. For very complicated flow problems – and ocean flows are certainly in this category – this is really the best that we can hope for. Indeed, in our view, this is the only way to proceed if the aim is to obtain a fundamental understanding of the processes involved. (We should add that, on completion of all such analyses, and having gained an appreciation of what each implies, it is useful to combine all in a suitable numerical simulation and thereby obtain some indication of how they all interact within the more complete solution. This, however, is not a worth-