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Study of the effects of stress sensitivity on the permeability and porosity of fractal porous media

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ABSTRACT

Flow in porous media under stress is very important in various scientific and engineering fields. It has been shown that stress plays an important role in effect of permeability and porosity of porous media. In this work, novel predictive models for permeability and porosity of porous media considering stress sensitivity are developed based on the fractal theory and mechanics of materials. Every parameter in the proposed models has clear physical meaning. The proposed models are evaluated using previously published data for permeability and porosity measured in various natural materials. The predictions of permeability and porosity show good agreement with those obtained by the available experimental data and illustrate that the proposed models can be used to characterize the flow in porous media under stress accurately.

1. Introduction

The stress sensitivity is defined as a phenomenon that the porous media is compressed. The permeability and porosity of porous media reduce as the stress increases. This phenomenon widely exists in both natural and artificial materials [1–3]. The study of flow through porous media under stress spans a broad range of science and engineering fields, including hydraulics [4,5], physics [6–9], chemical [10], petroleum and food engineering [11,12], etc. Tremendous attention also has been given to its methods, theories and progress in this area.

Numerical methods have played a significant role in the analysis of flow in porous media considering stress sensitivity. Kupková and Kupka [13] calculated the velocities of waves propagating through a system, which consisted of an elastic matrix with randomly distributed pores of uniform shapes with sizes governed by a power-law distribution. J. Larsson and R. Larsson, [14] researched the problem of coupled deformation and fluid diffusion in porous media based on the elastic–plastic solid theory. Schreyer-Bennethum [15] developed a theoretical foundation for flow and deformation in a highly interacting porous media, assuming that the porous material consists of a solid phase and liquid phase. Khoshghalb and Khalili [16] proposed a fully coupled meshfree algorithm to simulate the flow-deformation analysis of saturated porous media. The proposed model was effective in simulating the coupled flow deformation behavior in saturated porous media. Sabetamal et al. [17] represented the nonlinear behavior of the solid phase by either the Mohr Coulomb or Modified Cam Clay material model, and found that the contact interface between soil and structure is a crucial factor to avoid severe oscillations in the predicted dynamic response of the soil. However, the numerical methods demand tedious calculations and necessitate accurate models. These constraints limit the numerical methods' ability to describe the fluid flow in porous media under stress.

The conventional permeability models for stress sensitivity of porous media can be divided into two categories. One category is empirical models and another one is theory models.

Empirical permeability models include linear, exponential, power-law relationship functions, etc. Because the experiments of stress sensitivity are expensive, so a common practice is to look for linear permeability functions. A linear relationship makes interpolation easy and a line can be defined by only two measurements, which are a slope and an intercept term.

$$K = K_{\text{ref}}\left[c_1(p_{\text{eff}} - p_{\text{ref}}) + c_2\right] \tag{1}$$

where K is the permeability of porous media at the effective pressure p_{eff}, and K_{ref} is the permeability of porous media at the reference effective pressure p_{ref}.

The permeability of porous media considering the stress sensitivity also can be approximated by an exponential function [18].

$$K = K_{\text{ref}}\exp\left[-c_3(p_{\text{eff}} - p_{\text{ref}})\right] \tag{2}$$

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Riepe et al. [19], Debschutz et al. [20] and David et al. [21] showed that this empirical relationship was suitable for sandstones based on the laboratory experiment results. Evans et al. [22] also documented that the pressure-dependent permeability of rock with faults exhibits an exponential relationship.

Morrow et al. [23] provided a power-law function to describe the permeability changes of both clay-rich and non-clay reservoirs.

$$K = K_{\mathrm{ref}}(p_{\mathrm{eff}}/p_{\mathrm{ref}})^{c_4} \tag{3}$$

Shi and Wang [24] agreed that the relationship between effective pressure and permeability of crust should follow a power law based on the laboratory permeability measurement of fault gauge.

Jones [25] investigated the behavior of stress sensitive sandstones from low-permeability reservoirs which can be described by a cubic relationship.

$$K = K_{\mathrm{ref}}\left[1 - c_5 \ln(p_{\mathrm{eff}}/p_{\mathrm{ref}})\right]^3 \tag{4}$$

Ostensen [26] believed that the permeability of porous media was controlled by microcracks and confirmed that the permeability could be expressed by a square relationship function.

$$K = K_{\mathrm{ref}}\left[1 - c_6 \ln(p_{\mathrm{eff}}/p_{\mathrm{ref}})\right]^2 \tag{5}$$

Lei et al. [27] proposed that the relationship between intrinsic permeability and effective stress in reservoirs in general followed a quadratic polynomial functional form based on the results of experiments.

$$K = K_{\mathrm{ref}}\left(c_7 p_{\mathrm{eff}}^2 + c_8 p_{\mathrm{eff}} + c_9\right) \tag{6}$$

In Eqs. (1)–(6), the experimental parameters c_i $(i = 1, 2, \ldots, 8, 9)$ are fitted to experimental data. Because the empirical models cannot describe the microscopic deformation of porous media, they cannot explain the reason for the reduction in permeability for a porous media under stress.

Theoretical permeability models for the stress sensitivity of porous media have been usually developed for either a single capillary or pore. The different shapes of capillary or pore assumed by researchers lead to the different relationship between permeability and effective pressure of porous media.

The simplest representation of a pore is a cylindrical tube or spherical pore [28] and the relationship can be expressed as

$$K = \frac{r_0^2}{8}\left[1 - \frac{2(1-\nu^2)}{E} p_{\mathrm{eff}}\right]^2 \tag{7}$$

where r_0 is the radius of cylindrical tube or spherical pore at zero stress, ν is Poisson's ratio and E is Young's modulus.

The equation for an ellipse of large aspect ratio is similar to those for circles [29], but the equation for a thin ellipse depends upon the aspect ratio [30].

$$K = \frac{b_0^2}{8}\left[1 - \frac{2(1-\nu^2)}{\varepsilon E} p_{\mathrm{eff}}\right]^2 \tag{8}$$

where b_0 is the semi-minor width of pore and ε is the aspect ratio.

Gangi [31] devised phenomenological models to determine the variation with pressure of the permeability of whole and fractured porous rock. For whole porous rock, the permeability variation with pressure was based upon the Hertzian theory of deformation of spheres.

$$K = K_{\mathrm{ref}}\left[1 - C(p_{\mathrm{eff}}/p_{\mathrm{ref}})^{2/3}\right]^4 \tag{9}$$

where C is a constant that depends on the porous media.

For whole fractured rock, the permeability variation with confining pressure was determined by using a "bed of nails" model for the asperities of the fracture.

$$K = K_{\mathrm{ref}}\left[1 - (p_{\mathrm{eff}}/p_{\mathrm{ref}})^m\right]^3 \tag{10}$$

where m is a constant $(0 < m < 1)$ that characterizes the distribution function of the asperity lengths.

Using the same model for fluid flow through fractures with an exponential size distribution of asperities, Walsh [32] found that permeability varied as

$$K = K_{\mathrm{ref}}\left[1 - (\sqrt{2}h/a)\ln(p_{\mathrm{eff}}/p_{\mathrm{ref}})\right]^3 \tag{11}$$

where h is the root–mean–square height distribution of asperities, and a is the half width of the fracture aperture at the reference pressure p_{ref}.

The theoretical permeability models can illuminate the reduction of permeability in porous media to a certain extent. However, it is hard to describe the disordered and extremely complicated microstructures of porous media using theoretical permeability models that are developed for either a single capillary or pore.

Regarding the model for describing the relationship between effective stress and porosity, an exponential law has been developed for shale [33], carbonate [34] and sandstone [35]. The relationship can be written as follows.

$$\phi = \phi_{\mathrm{ref}} \exp\left[-c_{10}(p_{\mathrm{eff}} - p_{\mathrm{ref}})\right] \tag{12}$$

where ϕ is the porosity of porous media at the effective pressure p_{eff}, and ϕ_{ref} is the porosity of porous media at the reference effective pressure p_{ref}.

Hsu [36] found that the relationship follow a power law based on the results of experiments.

$$\phi = \phi_{\mathrm{ref}}(p_{\mathrm{eff}}/p_{\mathrm{ref}})^{c_{11}} \tag{13}$$

In Eqs. (12) and (13), the experimental parameters c_{10} and c_{11} are determined by fitting to experimental data.

Consequently, the characteristic behavior permeability and porosity of porous media under stress are still not well-determined. Because of the disordered and extremely complicated microstructures of porous media, it is difficult to find analytically the permeability and porosity of porous media under stress. So fractal theory is introduced to describe the microstructures of porous media. The flow behaviors in porous media have been studied extensively for more than two decades using fractal theory [37–43]. Pape et al. [44] presented a relation linking porosity to permeability based on a fractal model for the internal structure of the porous media. Good agreement between the predictions of the relation and the experimental data was obtained. Park et al. [45] developed a model based on fractal theory and incorporated a cake collapse effect to predict the porosity and permeability of the cake layers. Costa [46] derived a permeability-porosity equation using the classical Kozeny–Carman approach and a fractal pore-space geometry assumption. The equation was simple and described the permeabilities of different non-granular porous media. Tan et al. [47] analyzed the effect of stress on the permeability of porous media, considering the power-law stress-strain diagram of solid cluster. But it is believed that the Hook-law stress-strain diagram is more suitable for the research of porous media under the stress, based on experimental data. Further research is needed.

In this paper, we attempt to establish a theoretical model for permeability and porosity under stress based on the fractal theory and mechanics of materials. To this end, fractal characteristics of porous media considering stress sensitivity will be introduced in Section 2. Then fractal models for permeability and porosity under stress are derived in Section 3. Next, in Section 4, relevant results and discussions are demonstrated. Finally, conclusions will be given in Section 5.