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Multiple detectors "Influence Method"

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HIGHLIGHTS

- "Multiple Detector Influence Method" developed for uncertainty reduction.
- Absolute particle counting in absence of known detector efficiency.
- Detector set efficiency determination.

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ABSTRACT

The "Influence Method" is conceived for the absolute determination of a nuclear particle flux in the absence of known detector efficiency and without the need to register coincidences of any kind.

This method exploits the influence of the presence of one detector in the count rate of another detector, when they are placed one behind the other and define statistical estimators for the absolute number of incident particles and for the efficiency (Rios and Mayer, 2015a). Its detailed mathematical description was recently published (Rios and Mayer, 2015b) and its practical implementation in the measurement of a moderated neutron flux arising from an isotopic neutron source was exemplified in (Rios and Mayer, 2016).

With the objective of further reducing the measurement uncertainties, in this article we extend the method for the case of multiple detectors placed one behind the other. The new estimators for the number of particles and the detection efficiency are herein derived.

1. Introduction

The present work is motivated in the certain possibility of further reducing the statistical uncertainty with which a particle flux is measured through the previously introduced "Influence Method" (Rios and Mayer, 2015a, 2015b). Within the previous scheme, two detectors of similar but unknown efficiency were employed. The extension to different efficiencies and of particle out-scattering in the detector, were also treated in the original publication. Now we will extend the method to the case of a number of detectors involved in the measurement, greater than the original two. To develop the new scheme it is convenient to start with a brief description of the results previously obtained.

The "Influence Method" was initially conceived using two detectors with equal efficiency placed one after the other and considering the number of particles ($\boldsymbol{n}$) falling upon the detector during the counting time (Δt) to be a constant ($\boldsymbol{n}$=constant.) .This is the case where you want to measure a single event that generates $\boldsymbol{n_o}$ particles, such as a single burst of a Plasma Focus, a pulse of a Z-pinch experiment, an experiment of inertial confinent fusion (ICF), etc.

Let, in the simplest case, two detectors with the same efficiency ε, be placed one behind the other at a certain distance from the radiation source as schematized in Fig. 1. The number of particles counted by detector X is an aleatory variable ($\boldsymbol{X}$) whose distribution is a binomial of parameters $\boldsymbol{n}$ and $\boldsymbol{\varepsilon}$ ($X\sim Bi(n,\,\varepsilon)$). In the proposed scheme, particles not detected at X ($X_{out}=n-X$) impinge on detector Y. Thus, the number of those particles detected by Y are an aleatory variable ($\boldsymbol{Y}$) whose distribution is also a binomial of parameters $\boldsymbol{n}$ and $\varepsilon\cdot(1-\varepsilon)=\varepsilon\cdot q$ ($\boldsymbol{Y}\sim Bi(n,\,\varepsilon q)$), where $q=(1-\varepsilon)$ represents the probability of not being detected by X.

This scheme can be interpreted as a method where the sample of the second variable is influenced by the first one, for which reason we call it the "Influence Method". This influence manifests itself through the correlation between X and Y. Within this scheme we define an estimator for the population and an estimator for the efficiency as,.

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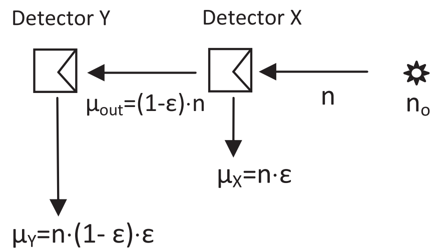

Fig. 1. Original measurement array scheme proposed by the "Influence Method".

$$\hat{n} = \frac{X^2}{X-Y} \qquad \hat{\varepsilon} = \frac{X-Y}{X}$$

As the estimators are non linear functions of two correlated variables, the calculation of uncertainties requires a certain algebraic effort which was developed in detail in (Rios and Mayer, 2015b), arriving at the expressions for the estimators with their uncertainties for a set of measurements (x,y) as follows:

$$\hat{n} = \frac{x^2}{x-y} \pm \left(\frac{xy\sqrt{(x+y)}}{(x-y)^2}\right) \qquad \hat{\varepsilon} = \frac{x-y}{x} \pm \left(\sqrt{\frac{y(x+y)}{x^3}}\right)$$

The condition for the valid application of the method is:

$$n \gg 2/\varepsilon^3 + 5/(\varepsilon \cdot (1-\varepsilon))$$

In the same reference, the treatment can be found for cases of detectors with different efficiencies and when some particles are removed from the beam by scattering in the detector.

The extension of the method for its application to a radioactive source, which adds its natural statistical emission fluctuations, was published in (Rios and Mayer, 2016). In this case, the number of particles falling on the forward detector (detector X), is represented by an aleatory variable (***Z***) Poisson distributed with parameter $\lambda = n$ ($\boldsymbol{Z \sim Poi(n)}$). Here n is the expected value of Z, the amount of particles incident upon the detector, so they relate to the number of source particles in the same time interval (n_o) through the geometrical efficiency (ε_g) through ($n = n_o \cdot \varepsilon_g$). Thus, detector X obeys a binomial distribution of parameters Z and ε ($X \sim Bi(Z, \varepsilon)$), being ε the intrinsic efficiency of that detector. In (Rios and Mayer, 2016) it is demonstrated that under these conditions X is also Poisson distributed ($\boldsymbol{X \sim Poi(n \cdot \varepsilon)}$) and that under these conditions, Y is also Poisson distributed ($\boldsymbol{Y \sim Poi(n \cdot \varepsilon \cdot q)}$). Since the variables are Poisson distributed, the expected values are the same as when the variables only obeyed binomial distributions, but the variances do change.

The effect of background (***B***) was also considered, yielding expressions with the corresponding uncertainties:

$$\hat{n} = \frac{(x-B)^2}{x-y} \pm \sqrt{\left(\frac{x^3 \cdot \left((x-2y)^2 + xy\right)}{(x-y)^4} + \frac{4 \cdot x^2}{(x-y)^2}\sigma_B{}^2\right)}$$

$$\hat{\varepsilon} = \frac{x-y}{(x-B)} \pm \sqrt{\left(\frac{y(x+y)}{x^3} + \frac{(x-y)^2}{x^4}\sigma_B{}^2\right)}$$

In this new work we present the "Multiple Detector Influence Method". It deals with the case where more than two detectors are placed successively one behind the other, with the aim to reduce overall statistical uncertainty, in case it can be implemented in practice. Estimators for the number of original particles and for the efficiency are deduced, along with the condition for the valid application of the method. We explicitly show how the uncertainty is reduced when the number of detectors is incremented, if the new condition of applicability is satisfied. Employment of this new development is useful when detectors of low efficiency are involved.

Section 2 deals with the case when the number of particles incident on the first detector is a constant (n=constant), applicable to an only burst of a pulsed source (Plasma Focus, Z-Pinch, ICF, etc) or, possibly in some case, to each time bin in a time-of-flight experiment.

Section 3 develops the results for the case of a radioactive source, a case where the particles incident on the first detector are therefore Poisson distributed.

The equations obtained are contrasted with Monte Carlo simulations to show their concordance.

2. Multiple detector Influence Method for n=constant

2.1. Estimator of the number of particles (n)

In this section we will deal with the case where the radiation particles falling upon the detector system are a constant number, as for instance one only radiation burst. This leaves out the Poisson distributed radiation sources.

Let us suppose that a number ***k*** of detectors possessing equal efficiency, are placed at a given distance from the source, one behind the other as depicted in Fig. 2. Under these conditions it was already demonstrated (Rios and Mayer, 2015a) that $X_1 \sim Bi(n, \varepsilon)$, $X_{out1} \sim Bi(n, q)$, $X_2 \sim Bi(n, \varepsilon q)$. Then, employing the same demonstration procedure as in (Rios and Mayer, 2015a, 2015b) it is easy to find the distribution corresponding to the rest of the variables shown in Table 1, where also, the expected values and variances of each variable are shown.

As it can be seen in the mentioned Fig. (2), $X_{out(j)}$ is what detector ***j*** did not detect. As a consequence, for any ***j***, the incident particles (***n***) can be written as:

$$n = X_1 + X_2 + X_3 + \ldots + X_j + X_{out(j)} \tag{1}$$

As a consequence, when ***k*** detectors are placed as above described, the number ***n*** can be estimated as the summation of all the measured numbers except the last two (***k-1*** and ***k***) and with the last two evaluate what was not detected by detector ***k-2*** by means of the application of the Influence Method.

$$\hat{n} = X_1 + X_2 + X_3 + \ldots + X_{(k-2)} + \hat{X}_{out(k-2)} \tag{2}$$

It must be noted that what was not detected by ***k-2*** ($X_{out(k-2)}$), is precisely what impinged on detector (***k-1***).

Then, with ***k*** successive detectors, the estimator for the number of particles is defined as:

$$\boldsymbol{\hat{n} = S + \hat{X}_{out(k-2)}} \tag{3}$$

Where the aleatory variable ***S*** is the summation of what was detected by all the detectors, except the last two,.

$$S = \sum_{i=1}^{(k-2)} X_i \tag{4}$$

Strictly for k > 2 and S=0 for k=2.

Utilizing the Influence Method, what was not detected by detector ***k-2*** can be estimated as:

$$\hat{X}_{out(k-2)} = \frac{X_{(k-1)}{}^2}{\left(X_{(k-1)} - X_k\right)} \tag{5}$$

To simplify the notation, we shall call : $X_o = X_{out(k-2)}$, $X = X_{(k-1)}$, $Y = X_k$, so in this way: