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Evaluation of the Hubbell rectangular source integral using Haar wavelets method

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HIGHLIGHTS

- A numerical integration method using Haar wavelets is detailed.
- Hubbell rectangular source integral is estimated with Haar wavelets method.
- Convergence study of the Haar wavelet method is performed.
- Comparison with earlier results is presented.

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ABSTRACT

Haar wavelets numerical integration method is exposed and used for evaluating the Hubbell rectangular source integral. The method convergence is studied to get the minimum iteration number for a desired precision. Haar wavelets results are finally compared to those obtained with other integration methods.

1. Introduction

The study of radiation field from a rectangular source leads to the resolution of the integral

$$H(a,b)=\int_0^b \arctan\left(\frac{a}{\sqrt{1+x^2}}\right)\frac{1}{\sqrt{1+x^2}}\,dx$$

known as the Hubbell rectangular source integral (Hubbell et al., 1960). Since there is no direct solution for this integral, many numerical methods were developed by Murley and Saad (2011), Ezure (2005), Guseinov et al. (2004) and Timus (1993). In this paper, Haar wavelets (Strang, 1989; Chen and Hsiao, 1997; Aziz and Haq, 2010) are used to evaluate the integral $H(a,b)$ in a simple way with a good accuracy and a very low time cost.

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2. Numerical integration using Haar wavelets

The discrete Haar wavelets family is constructed using a scaling function $\psi_{0,0}(x)$:

$$\psi_{0,0}(x)=\begin{cases}1 & \text{if } x\in[0,1)\\ 0 & \text{otherwise}\end{cases}$$

and the mother function $\psi_{1,0}(x)$ defined as follows:

$$\psi_{1,0}(x)=\begin{cases}1 & \text{if } x\in\left[0,\frac{1}{2}\right)\\ -1 & \text{if } x\in\left[\frac{1}{2},1\right)\\ 0 & \text{otherwise}\end{cases} \tag{1}$$

Then the other Haar wavelets functions $\psi_{j,k}(x)$ are generated using dilation (parameter j) and translation (parameter k)

$$\psi_{j,k}(x)=2^{j/2}\psi_{1,0}(2^jx-k)$$

The explicit form of functions $\psi_{j,k}(x)$ is

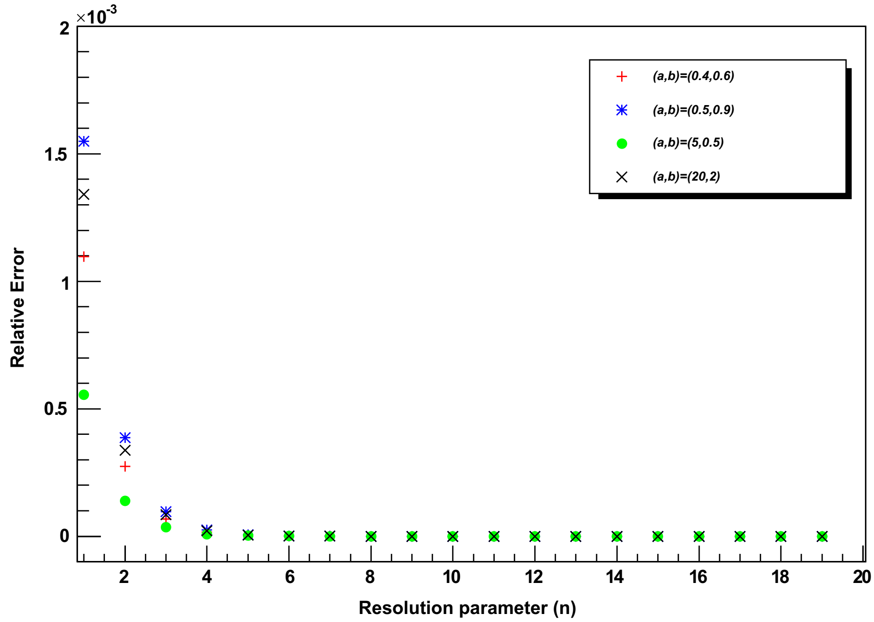

Fig. 1. Relative error versus resolution parameter (n).

$$\psi_{j,k}(x) = \begin{cases} 2^{j/2} & \text{if } x \in \left[a_{j,k}, \frac{a_{j,k}+b_{j,k}}{2}\right) \\ -2^{j/2} & \text{if } x \in \left[\frac{a_{j,k}+b_{j,k}}{2}, b_{j,k}\right) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where:

- $j = 2, 3, \ldots, 2^n$ is the resolution (dilation) parameter.
- $n = 1, 2, 3\ldots$ is the maximum level resolution.
- $k = 0, 1, \ldots, 2^j - 1$ is the translation parameter.
- $a_{j,k} = \frac{k}{2^j}$
- $b_{j,k} = \frac{k+1}{2^j}$

The set of Haar wavelets functions $\psi_{j,k}(x)$ forms an orthonormal basis of $L^2([0, 1))$ (Strichartz, 1993) and any function $f(x)$ in $L^2([0, 1))$ can be expanded by a Haar series of infinite terms:

$$f(x) = \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{j,k}\psi_{j,k}(x) \tag{3}$$

With taking finite terms as approximation, we obtain

$$f(x) \approx \sum_{j=0}^{2^n} \sum_{k=0}^{2^j-1} c_{j,k}\psi_{j,k}(x) \tag{4}$$

which gives an approximation of the integral:

$$\int_0^1 f(x)\,dx \approx \sum_{j=0}^{2^n} \sum_{k=0}^{2^j-1} c_{j,k} \int_0^1 \psi_{j,k}(x)\,dx \tag{5}$$

The double sum in Eq. (5) can be reduced to one term since

$$\int_0^1 \psi_{j,k}(x)\,dx = 0 \quad \text{for } (j, k) \neq (0, 0) \tag{6}$$

Thus the approximate integral (5) is

$$\int_0^1 f(x)\,dx \approx c_{0,0} \tag{7}$$

To get the Haar coefficient $c_{0,0}$, grid points $x_i = \frac{2i-1}{2^{n+2}}$, $i = 1, 2, \ldots, 2^{n+1}$, are taken and we get the formula (Aziz and Haq, 2010):

$$c_{0,0} = \frac{1}{2^{n+1}} \sum_{i=1}^{2^{n+1}} f(x_i) \tag{8}$$

3. Evaluation of the Hubbell rectangular source integral using Haar wavelets integration method

Formula (7) applied to the integral $H(a, b)$ after substituting $u = x/b$ gives the following approximation:

$$H(a, b) \approx H(a, b, n) = \frac{b}{2^{n+1}} \sum_{i=1}^{2^{n+1}} \arctan\left(\frac{a}{\sqrt{1+\left(\frac{b(2i-1)}{2^{n+2}}\right)^2}}\right) \frac{1}{\sqrt{1+\left(\frac{b(2i-1)}{2^{n+2}}\right)^2}} \tag{9}$$

3.1. Convergence study of Haar wavelets method

The quantity $H(a, b, n)$ was estimated using ROOT (root.cern.ch), a free object-oriented data analysis framework based on C++ provided by CERN. To study the convergence of the method, the integral is estimated for different values of (a, b) and for $n = 1, 2, 3, \ldots, 20$. $H(a, b, n)$ is then compared to $H(a, b, 30)$ which can be considered as the exact value of $H(a, b)$ since $\lim_{2^n \to +\infty} H(a, b, n) = H(a, b)$.

The relative error is defined as follows:

$$\Delta(a, b, n) = \frac{|H(a, b, n) - H(a, b, 30)|}{H(a, b, 30)} \tag{10}$$