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Comment on “Exact explicit travelling wave solutions for (n+1)-dimensional Klein–Gordon–Zakharov equations” [Chaos, solitons and fractals 34(2007) 867–871]

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ABSTRACT

Jibin Li studied “exact explicit travelling wave solutions for (n+1)-dimensional Klein–Gordon–Zakharov equations.” By using the approach of dynamical system, the author claimed that they had obtained five classes of exact explicit travelling wave solutions. However, we checked the results and found two typographical technical errors in the main results. Furthermore, the author fails to note, for $a = 0, b < 0, h \in (0, \infty)$ and $a < 0, b > 0, h = 0$, there is a family of periodic wave solutions and a series of breaking wave solutions, respectively.

1. Introduction

In [1], Jibin Li studied the following (n+1)-dimensional Klein–Gordon–Zakharov equations

$$\begin{aligned}&\phi_{tt} - \triangle\phi + \phi = -\phi\psi,\\ &\psi_{tt} - c^2\psi = \triangle|\phi|^2,\end{aligned} \tag{1.1}$$

where $\triangle$ is the Laplacian operator and c is the wave's propagation speed. Eq. (1.1) describe the interaction of the ion acoustic wave and the Langmuir wave in a plasma, and are an important class of coupled nonlinear wave model. The function ψ is the ion density's deviation from its equilibrium and the function ϕ is the electric field's fast time scale component raised by electrons. In order to find exact solutions of Eq. (1.1), the author assumed that

$$\begin{aligned}&\phi(x,t) = u(\xi)e^{i\eta},\ \psi(x,t) = v(\xi),\\ &\eta = \sum_{j=1}^{n}\alpha_j x_j + \beta t,\ \xi = \sum_{j=1}^{n}\gamma_j x_j - \sigma t, x \in R^n, t \in R,\end{aligned} \tag{1.2}$$

where $\alpha_j (j = 1 \ldots n), \beta, \gamma_j (j = 1 \ldots n)$ and σ are travelling wave parameters. Through a series of transformations, Eq. (1.1) were reduced to the following two-dimensional Hamiltonian system

$$\frac{du}{d\xi} = y, \quad \frac{dy}{d\xi} = au + bu^3, \tag{1.3}$$

with the Hamiltonian

$$H(u, y) = -\frac{a}{2}u^2 - \frac{b}{4}u^4 + \frac{1}{2}y^2 = h, \tag{1.4}$$

where $a = \frac{\sum_{j=1}^{n}\alpha_j^2 + 1 - \beta^2}{\sum_{j=1}^{n}\gamma_j^2 - \sigma^2}$, $b = \frac{\sum_{j=1}^{n}\gamma_j^2}{(\sum_{j=1}^{n}\gamma_j^2 - \sigma^2)(\sigma^2 - c^2\sum_{j=1}^{n}\gamma_j^2)}$, $\sum_{j=1}^{n}\alpha_j\gamma_j + \beta\sigma = 0$ and h is the integral constant.

In [1], the author considered the dynamics of the phase orbits of system (1.3) in the phase plane (u, y) and solved Eq. (1.1). But when we checked the results, we found that the exact solutions correspond to homoclinic orbits and heteroclinic orbits in [1] were incorrect. In this paper, we reconsider the dynamics of the phase orbits of system (1.3) and give the correct exact solutions of Eq. (1.1). Here, we always suppose that $b \neq 0$. Otherwise, the system (1.3) will be linear system. With the aid of Maple software, the system (1.3) has different bifurcations of phase portraits are shown in Figs. 1–4.

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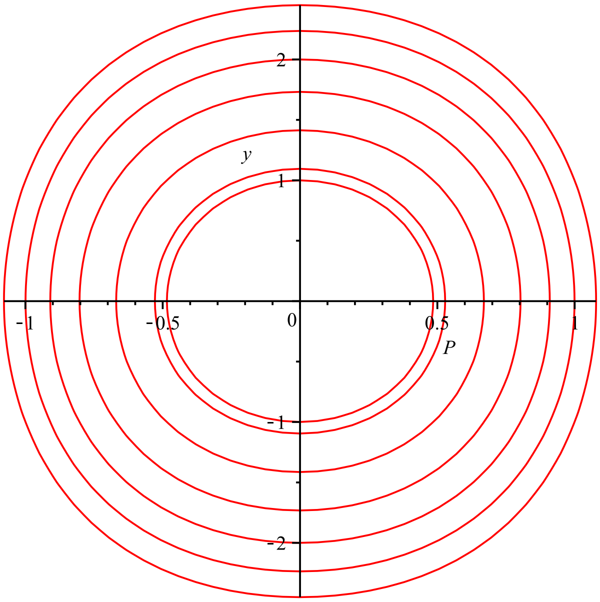

Fig. 1. The bifurcations of phase portraits of (1.3) ($a \le 0$, $b < 0$).

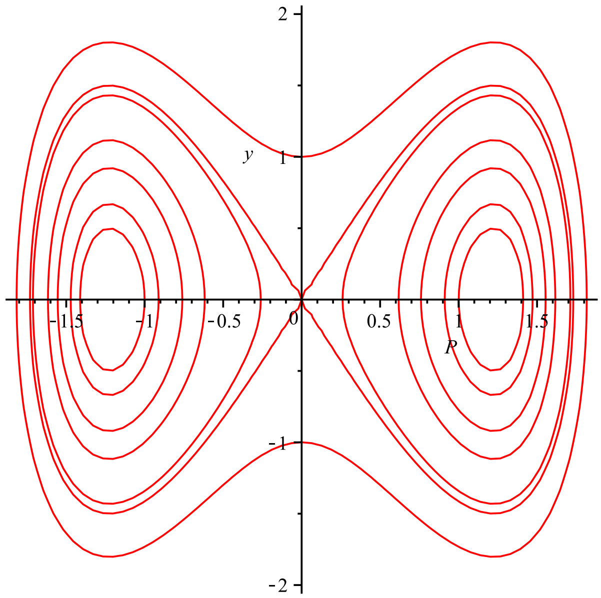

Fig. 3. The bifurcations of phase portraits of (1.3) ($a > 0$, $b < 0$).

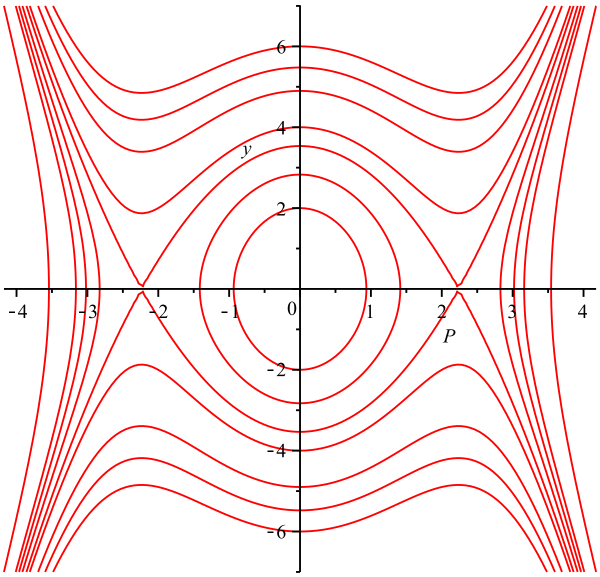

Fig. 2. The bifurcations of phase portraits of (1.3) ($a < 0$, $b > 0$).

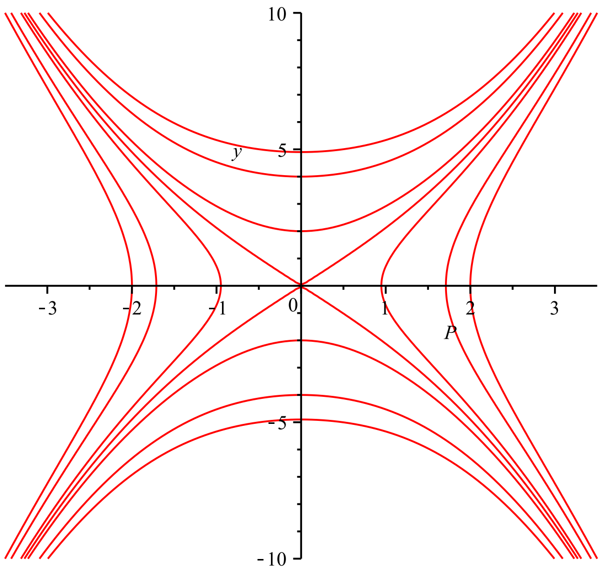

Fig. 4. The bifurcations of phase portraits of (1.3) ($a \ge 0$, $b > 0$).

2. The exact solutions of system (1.3)

According to the above phase portraits for system (1.3) , by using the theory of bifurcations and Jacobian elliptic functions [2], we obtain the following results:

(1) When $a < 0$, $b < 0$ (see Fig. 1), the system (1.3) has an unique equilibrium. The origin $O(0,0)$ is a centre point. There is a family of periodic orbits of system (1.3). It follows from (1.4) that $y^2 = \frac{-b}{2}(\frac{4h}{-b} - \frac{2a}{b}u^2 - u^4) = \frac{-b}{2}(z_1^2 + u^2)(z_2^2 - u^2)$, where $z_1^2 = \frac{1}{-b}(-a + \sqrt{a^2 - 4hb})$, $z_2^2 = \frac{1}{-b}(a + \sqrt{a^2 - 4hb})$. As $h \in (0, \infty)$, the family of periodic orbits defined by $H(u, y) = h$ of (1.4) gives rise to a family of periodic wave solutions

$$u(\xi) = z_2 cn\left(\sqrt{\frac{-b(z_1^2 + z_2^2)}{2}}\xi, \frac{z_2}{\sqrt{z_1^2 + z_2^2}}\right). \tag{2.1}$$

(2) When $a = 0$, $b < 0$ (see Fig. 1), the system (1.3) has an unique equilibrium. The origin $O(0,0)$ is a centre point. There is a family of periodic orbits of system (1.3). It follows from (1.4) that $y^2 = \frac{-b}{2}(\sqrt{\frac{4h}{-b}} - u^2)(\sqrt{\frac{4h}{-b}} + u^2)$. As $h \in (0, \infty)$, the family of periodic orbits defined