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Optimal control and stability analysis of an epidemic model with population dispersal

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ABSTRACT

In the present paper we consider an SEIR type epidemic model with transport related infection between two cities. It is observed that transportation among regions has a strong impact on the dynamic evolution of a disease which can be eradicated in the absence of transportation. Transportation can lead to the incorporation of a positive risk probability. The epidemiological threshold, commonly known as the basic reproduction number, is derived and it is observed that when the basic reproduction number is less than unity the disease dies out, where as if it exceeds unity the disease may persist in the system. A thorough dynamical behavior of the constructed model is studied. We formulate and solve an optimal control problem using vaccination as a control tool. Extensive numerical simulations are carried out based on our analytical results. Finally we try to relate our work with a real world problem.

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1. Introduction

Mathematical modeling is a very effective tool to study and analyze different epidemiological problems. In epidemiology models allow us to predict the population level epidemic dynamics from an individual level knowledge of epidemiological factors, the long term behavior from the early invasion dynamics, or the impact of vaccination on the spread of infection. Though there is a long and distinguished history of mathematical modeling in epidemiology, however, it was not until the early 1990s that the increasingly popular dynamical systems approaches where applied to epidemiology. Some interesting and important development can be found in [8,10,14–16,18].

With the expansion of computing, there has been an explosion in the development and use epidemiological models, however, deficiencies still persist in the use of mathematical models. A mathematical model on epidemic problem would be more realistic if population dispersal is considered. SARS could be a good example of infectious disease, which was broke out in almost all parts of China and some other parts of the world only due to people's dispersal. Takeuchi et al. [27], Wang and Wang [30], Wang and Mulone [31], Wang and Zhao [32,33] proposed and analyzed epidemic models to describe the dynamical behavior of disease spread between multiple patches. Arino and Vanden Driessche [1], Wan and Cui [29] proposed multicity epidemic models to study the dynamics of infectious diseases. Liu and Takeuchi [21] considered a transport related epidemic model with entry screenings. In a later work, Liu et al. [20] considered another transport related epidemic model with both entry and exit screenings. Arino and Driessche [1] investigated the effect of population dispersal among n patches on the spread of a disease. Their numerical simulations show that, if a disease can extinct in two patches when they are isolated, it can be still extinct within two patches when dispersal rates of individuals are in some range, otherwise it will

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be endemic. Their simulation also shows that, if the basic reproduction numbers of isolated patches are not very large and the contact rates in two patches are not very large too, population dispersal may help eradicate the diseases which can be endemic in either patch when they are isolated. All the above works ignore the possibility for the individuals to become infective during travel. Allen et al. [2] first proposed a SIS epidemic model including transport related infection on disease spread. Recently, Chen et al. [4] also proposed an SIR model with transport related infections. Inspite of the above stated articles epidemic models with population dispersal needs to be further research. From this view point, in our present work, we consider an epidemic model with population dispersal from one place to another. For simplicity, we consider only the population dispersal between two cities.

Presently different control measures are applied to improve control and ultimately to eradicate the infection from the population. Among all the control measures vaccination is one of the most important and effective controls. It acts by simulating a host immune response such that immunized individuals are protected against infection. Some works using vaccination control can be found in [15,16,23,28,34]. Kar and Jana describe an SIRV model to minimize the infected population as well as the costs require to control the disease by using vaccination and treatment. Rodrigues et al. [26] investigated the optimal vaccination strategy for the dengue epidemic considering both the costs of treatment of infected individuals and the costs of vaccination.

In the present article we consider an epidemic model with population dispersal and vaccination as a control measure. Vaccination operates by reducing the number of susceptible individuals in the population. Further one of the important phenomena in epidemic model is the form of disease transmission. In most of the articles, authors consider disease transmission in a bilinear form. But the main drawback of the bilinear type force of infection rate is, if either of susceptible or infected population goes to higher range, then the disease transmitted in a huge rate compare to the original rate. To overcome this drawback, we consider here the disease transmission in a saturated form. For more details about the saturation form, one may read the articles by Zhang et al. [35], Chong et al. [5] and references therein.

The paper is organized as follows: In Section 2 we describe the model assumptions and the corresponding mathematical model. The model is analyzed in Section 3. In Section 4 we construct an optimal control problem and solve it by considering vaccination as the control. Some simulation works are illustrated in Section 5, and a brief conclusion is given in the last section.

2. Model formulation

In this section we formulate a SEIR epidemic model with population dispersal between two cities. The model is formulated with the state variables S_i, E_i, I_i, R_i and $N_i \quad (i = 1, 2)$ that represent respectively, the number of susceptible, exposed, infected, recovered individuals and total population in city i, where $N_i = S_i + E_i + I_i + R_i$ for $i = 1, 2$. For simplicity we consider that both the cities are identical in the sense of population density, economics, medical services, living condition and disease transmission probability etc. This enables us to consider the parameters associated with the system are identical for both the cities.

To construct the model we make the following assumptions.

- A is the total recruitment at any time t. Among them $u(t)$ portion is vaccinated and hence goes to the recovered. The rest portion remains to the susceptible class. We also assume that a susceptible individual goes into the infected class after being infected through the exposed class.
- For the transmission of disease, saturated incidence rate of the form $\frac{\beta S_i I_i}{1+\eta I_i}\,(i = 1,\ 2)$ is considered. β is the maximum contact rate between susceptible and infected individuals or known as a parameter for the force of infection and η is the reciprocal of half saturation constant. In this incidence rate the number of effective contacts between the infected and susceptible individuals may saturate at high infecting levels due to crowding effect of the infected individuals or due to behavioral change of the susceptible individuals. It also prevents the unboundedness of the contact rates (see [35]).
- There is a direct transport between two cities. Per capita dispersal rate of individuals of every city i to city j $(j \neq i, i, j = 1, 2)$ is taken as α.
- When the individuals in city j travel to city i, disease is transmitted with the incidence rate $\frac{\gamma\alpha S_j I_j}{1+\eta I_j}$, $j = 1, 2$ with a transmission rate $\gamma\alpha$, and that portion of individual goes to the exposed class from susceptible class (γ is the transport related disease transmission rate and $j \neq i,\ \ i, j = 1, 2$).
- ρ is the per capita rate by which exposed individuals become infected individuals. It is evident that sometimes the pathogen of an infectious disease enables a person exposed for disease but due to high immunity power and the immune system producing antibodies against antigens enabling to destroy the harmful substances, they can recover without being infected. Therefore, we assume that a portion of the exposed individual directly moves to the recovered class without being infected.
- Natural death rate for each class of individuals of both the cities are taken as d. Further we assume that infected individual's per capita death rate is δ.
- The recovery rate of infected individual is taken as b.
- After recovery from the disease, some portion of the individuals may be susceptible for the disease and this portion is taken as σ.
- The individuals have no birth or death during their travel.
- Susceptible individuals become infected during travel (see [2,4]).

Following the above assumptions we construct a transport related SEIRS epidemic model given as:

$$\begin{aligned}\frac{dS_1}{dt} &= A(1-u) - \frac{\beta S_1 I_1}{1+\eta I_1} - dS_1 + \sigma R_1 \\ &\quad -\alpha S_1 + \alpha S_2 - \frac{\gamma\alpha S_2 I_2}{1+\eta I_2},\\ \frac{dE_1}{dt} &= \frac{\beta S_1 I_1}{1+\eta I_1} - (d+\rho+\xi+\alpha)E_1 + \alpha E_2 + \frac{\gamma\alpha S_2 I_2}{1+\eta I_2},\end{aligned}$$

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