Chaos, Solitons and Fractals 84 (2016) 9–14

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Scaling and nonlinear behaviour of daily mean temperature time series across India

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A R T I C L E   I N F O

Article history:
Received 7 October 2015
Accepted 17 December 2015
Available online 12 January 2016

Keywords:
Hurst Exponent
Daily mean temperature
Detrended Fluctuation Analysis
Higuchi
Visibility Graph Analysis
Delay Vector Variance

A B S T R A C T

In order to ascertain the dynamics of temperature variation in India, the scaling properties of the daily mean temperature time series obtained from seven different weather stations viz. Kolkata, Chennai, New Delhi, Mumbai, Bhopal, Agartala and Ahmadabad representing different geographical zones in India has been studied. Scaling properties of the temperature profile across India has been estimated from the calculation of Hurst-Exponent parameter obtained from five different scaling methods. Hurst Exponent values confirm that all temperature time series are Fractional Brownian Motion (FBM), statistically self-affine, anti-persistent and Short Range Dependent (SRD) self similar. As SRD self similarity is a common signature of a nonlinear dynamical process, further investigation has been made to discover the presence of any nonlinear behaviour of the temperature profile of Indian climate using Delay Vector Variance (DVV) method and the present calculation confirms a deterministic nonlinear profile of the same.

1. Introduction

The weather in India is as diverse as the country itself. India is a country that is spread along vast physical distance, with mountains in the north, plains in the centre and the vast seas in the south. The wide variations in climate and weather conditions in India are due to the varied topology and large geographical area of the country. As India lies in the south of the Asian sub-continent it is surrounded by the Bay of Bengal from east, the Arabian Sea from the west and the vast and almost endless Himalayan mountain range from the other side. This vast country and diverse climatic conditions have helped India to become one of the major agricultural countries. The changing weather from region to region has helped India in producing different kinds of vegetations under different climatic conditions. On the other hand, the uncertainty of weather conditions in India puzzles the geographers and agriculturists about the onset of the monsoon rains, the unpredictable fluctuations in rainfall, the sudden flooding, the rapid erosion, the extremes of temperature and the tropical storms [1,2]. The wide difference of climate in India can be adjudged from a complex analysis of different weather parameters of different geographical areas. Proper analysis demands more sensible techniques and hence for the last few decades various advanced statistical or numerical methods has been developed which can help to understand the dynamics of the weather in India

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http://dx.doi.org/10.1016/j.chaos.2015.12.016
0960-0779/

[3]. As temperature is one of the prime contributors for climate variation, the daily mean temperature data [4] which is basically discrete time signal or time series [5] recorded (from 9th October, 1996 to 1st February, 2013) at seven different stations viz. Kolkata, Chennai, New Delhi, Mumbai, Bhopal, Agartala and Ahmadabad representing different geographical zones of India has been analysed in this work to understand the dynamics (scaling behaviour and nonlinearity) of temperature profile of the country.

Noise is extraneous information in a signal that needs to be filtered out and in this work Discrete Wavelet Transform (DWT) [6,7] has been used to de-noise the temperature time series under investigation. In order to reveal the statistical character of the signals with respect to different scales and also to identify the behaviour of the signals with respect to signal fluctuation (Fractional Brownian Motion [8,9] or fractional Gaussian noise [10], stationary or non-stationary) the Hurst-Exponents [11,12] has been estimated. Though various methods are very widely used to estimate the Hurst Exponent, owing to their merits, they also suffer from some serious limitations. So applying a single method to calculate the Hurst Exponent is not sufficient in making indisputable conclusion about the scaling property of the temperature time series obtained from the seven weather stations across India. Here, five different methods like General method [13], Detrended Fluctuation Analysis (DFA) [14–17], Wavelet Variance Analysis (WVA) method [18–21], Higuchi's method [22], Visibility Graph Analysis [12,23,24] and has been chosen to calculate the Hurst Exponent and thus confirm adequate authenticity about the outcome of the analysis. Delay Vector Variance (DVV) [25–27] has been applied on the data to investigate the presence of nonlinear dynamics in the time series.

2. Methodology

2.1. DWT denoising

The time–frequency decomposition ability of the wavelet technique is the prime attribute that is used in atmospheric application. The reason to choose Discrete Wavelet Transform (DWT) over continuous wavelet transform (CWT) is that in CWT, most of the information related to close scales or times is redundant and it yields a high computational cost. To avoid redundancies, those wavelet functions are chosen that form an orthogonal basis. Discrete Wavelet Transform uses discrete values of scale (j) and localisation (k).

Out of the many existing mother wavelets (orthogonal bases) like Daubechies, Coiflet, Symmlet and Haar wavelet [28], the most suitable mother wavelet for a particular signal is chosen based on minimising the entropy of the wavelet transformed matrix [29,30]. An entropy score for each transform is computed, and the mother wavelet that produces the best score is picked. If D is the DWT of the data using a particular mother wavelet then Shannon entropy measure is given by

$$h(D) = -\sum\nolimits_{j,k} d_k^{j'} \log d_k^{j'}, \tag{1}$$

where $d_k^{j'}$ are the nonnegative normalised wavelet coefficients, that is, $d_k^{j'} = |d_k^j| / \sum |d_k^j|$ and $0 \log 0 = 0$ by definition. The goal is to determine the mother wavelet for which the value of $h(D)$ will be minimum. Table 1 shows the best possible choices of mother wavelets for temperature data of different stations.

To remove insignificant information wavelet shrinkage is performed, a process of shrinking the coefficients of D by setting it to zero or shrinking it towards certain threshold coefficients. The threshold value λ, known as universal threshold, is obtained as [31]

$$\lambda_n^u = (2 \log n)^{1/2}\, \hat{\sigma}, \tag{2}$$

where n is the number of data points, $\hat{\sigma}$ is an estimate of the noise level σ (typically a scaled median absolute deviation of the empirical wavelet coefficients).

2.2. Estimation of Hurst-Exponent

Hurst parameter values estimated from different methods are of different forms and yield important insights about the scaling behaviours of the temperature signals studied here. The value of Hurst parameter H lies between 0 and 1. Depending upon different values of H, different inference can be made about the time series. As for example, if $0 < H < 0.5$, the time series is antipersistent and has short range dependent memory. If $H = 0.5$, the time series is random whereas if $0.5 < H < 1$ the time series is persistent with long range dependent memory. A generalised approach proposed by Hurst [11], Detrended Fluctuation Analysis (DFA) approach proposed by Peng et al. [14] and Chen et al. [32], Higuchi's method [22] and Wavelet Variance Analysis method [18–20] are all very familiar methods and have been applied widely and satisfactorily in climatic data analysis. But the method of Visibility Graph Analysis is a comparatively new method [23,33] and it requires a brief explanation.

In the generalised approach, for a time series $X(t)$, (where $t = \nu, 2\nu, \ldots k\nu, \ldots, T$) with the observation period (T) and the time resolution (ν), the Hurst Exponent $H(m)$ is related to the m-order moments $Z_m(\partial)$ with an increment of ∂ of the data series [34,35] by the following relation

$$Z_m(\partial) \sim \left(\frac{\partial}{\nu}\right)^{mH(m)} \tag{3}$$

The Hurst exponent H is estimated when $m = 1$ i.e, $H(1)$.

In Detrended Fluctuation Analysis (DFA) [14] approach, the DFA exponent h is related to $F(s)$ as

$$F(s) \sim s^h \tag{4}$$

where $F(s)$ is the variance of the local trend of the data series with a scale size of s.

The Hurst Exponent can be estimated as

$$H = \begin{cases} h & (for \quad Stationary) \\ h-1 & (for \quad non\text{-}Stationary) \end{cases} \tag{5}$$

Wavelet has been used in various methods like that described by Murgia et al. [36] and Chamoli et al. [37], to estimate the fractal properties and hence Hurst Exponent