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Remarks on estimating upper limit of norm of delayed connection weight matrix in the study of global robust stability of delayed neural networks

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Abstract

The question of estimating the upper limit of the norm $\|\boldsymbol{B}\|_2$ of the delayed connection weight matrix $\boldsymbol{B}$, which is a key step in some recently reported global robust stability criteria for delayed neural networks (DNNs), is considered. An estimate of the upper limit of $\|\boldsymbol{B}\|_2$ was previously given by Cao, Huang and Qu. More recently Singh has presented an alternative estimate. Presently it is shown that an estimate of the upper limit of $\|\boldsymbol{B}\|_2$ may be found in some cases, which would be an improvement over each of the above-mentioned two estimates. Some observations concerning the determination of the least conservative upper limit of $\|\boldsymbol{B}\|_2$ are presented.

1. Introduction

Consider the delayed neural network (DNN) model defined by the following state equations [1–42]:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{x}(t-\tau)) + \boldsymbol{u} \tag{1}$$

or

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau)) + u_i \quad i = 1, 2, \ldots, n, \tag{2}$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is the state vector associated with the neurons, $\boldsymbol{C} = \mathrm{diag}\,(c_1, c_2, \ldots, c_n)$ is a positive diagonal matrix ($c_i > 0$, $i = 1, 2, \ldots, n$), $\boldsymbol{A} = (a_{ij})_{n\times n}$ and $\boldsymbol{B} = (b_{ij})_{n\times n}$ are the connection weight and the delayed connection weight matrices, respectively, $\boldsymbol{u} = [u_1, u_2, \ldots, u_n]^{\mathrm{T}}$ is a constant external input vector, τ is the transmission delay, the f_j, $j = 1, 2, \ldots, n$, are the activation functions, $\boldsymbol{f}(\boldsymbol{x}(\cdot)) = [f_1(x_1(\cdot)), f_2(x_2(\cdot)), \ldots, f_n(x_n(\cdot))]^{\mathrm{T}}$, and the superscript 'T' to any vector (or matrix) denotes the transpose of that vector (or matrix). It is understood that the activation functions satisfy the following restrictions

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$$|f_j(\xi)| \leqslant M_j \quad \forall \xi \in R; \quad M_j > 0, \quad j = 1, 2, \ldots, n$$

and

$$0 \leqslant \frac{f_j(\xi_1) - f_j(\xi_2)}{\xi_1 - \xi_2} \leqslant L_j \quad j = 1, 2, \ldots, n$$

for each ξ_1, $\xi_2 \in R$, $\xi_1 \neq \xi_2$, where L_j are positive constants. The quantities c_i, a_{ij}, and b_{ij} may be considered as intervalized as follows:

$$\begin{aligned} \boldsymbol{C}_I &:= [\underline{\boldsymbol{C}}, \overline{\boldsymbol{C}}] = \{\boldsymbol{C} = \mathrm{diag}(c_i) : \underline{\boldsymbol{C}} \leqslant \boldsymbol{C} \leqslant \overline{\boldsymbol{C}}, \text{i.e.}, \underline{c}_i \leqslant c_i \leqslant \overline{c}_i, i = 1, 2, \ldots, n\}, \\ \boldsymbol{A}_I &:= [\underline{\boldsymbol{A}}, \overline{\boldsymbol{A}}] = \{\boldsymbol{A} = (a_{ij})_{n\times n} : \underline{\boldsymbol{A}} \leqslant \boldsymbol{A} \leqslant \overline{\boldsymbol{A}}, \text{i.e.}, \underline{a}_{ij} \leqslant a_{ij} \leqslant \overline{a}_{ij}, i, j = 1, 2, \ldots, n\}, \\ \boldsymbol{B}_I &:= [\underline{\boldsymbol{B}}, \overline{\boldsymbol{B}}] = \{\boldsymbol{B} = (b_{ij})_{n\times n} : \underline{\boldsymbol{B}} \leqslant \boldsymbol{B} \leqslant \overline{\boldsymbol{B}}, \text{i.e.}, \underline{b}_{ij} \leqslant b_{ij} \leqslant \overline{b}_{ij}, i, j = 1, 2, \ldots, n\}. \end{aligned} \tag{3}$$

Definition 1. The system given by (1) with the parameter ranges defined by (3) is globally robust stable if the unique equilibrium point $\boldsymbol{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]^{\mathrm{T}}$ of the system is globally asymptotically stable for all $\boldsymbol{C} \in \boldsymbol{C}_I$, $\boldsymbol{A} \in \boldsymbol{A}_I$, $\boldsymbol{B} \in \mathbf{B}_I$.

In the following, if $\boldsymbol{H}$ is a matrix, its norm $\|\boldsymbol{H}\|_2$ is defined as

$$\|\boldsymbol{H}\|_2 = \sup\{\|\boldsymbol{Hx}\| : \|\boldsymbol{x}\| = 1\} = \sqrt{\lambda_{\max}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})},$$

where $\lambda_{\max}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})$ denotes the maximum eigenvalue of $\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}$. In [9] it has been shown that

$$\|\boldsymbol{B}\|_2 \leqslant (\|\boldsymbol{B}^*\|_2 + \|\boldsymbol{B}_*\|_2), \tag{4}$$

where $\boldsymbol{B}^* = (\overline{\boldsymbol{B}} + \underline{\boldsymbol{B}})/2, \boldsymbol{B}_* = (\overline{\boldsymbol{B}} - \underline{\boldsymbol{B}})/2$. The estimate (4), which is a modified and corrected version of the estimate given in [1], has been utilized in some recently reported global robust stability criteria (for example, [9–11,32,42]). More recently [37] an alternative estimate in the form

$$\|\boldsymbol{B}\|_2 \leqslant \|\boldsymbol{Q}\|_2 \tag{5}$$

has been presented, where $\boldsymbol{Q} = (q_{ij})_{n\times n}$ is defined by

$$q_{ij} = \max\{|\underline{b}_{ij}|, |\overline{b}_{ij}|\} \quad i, j = 1, 2, \ldots, n. \tag{6}$$

As shown in [37], in some cases (5) may be a better estimate than (4).

The purpose of this paper is to show that an estimate of the upper limit of $\|\boldsymbol{B}\|_2$ may be found in some cases, which would be an improvement over both (4) and (5). Some observations concerning the determination of the optimum (i.e., least conservative) upper limit of $\|\boldsymbol{B}\|_2$ are presented.

2. Possible new estimates of upper limit of $\|\boldsymbol{B}\|_2$

Consider a specific example [37] of a second-order DNN with

$$\overline{\boldsymbol{A}} = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}, \quad \underline{\boldsymbol{A}} = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix}, \quad \overline{\boldsymbol{B}} = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix}, \quad \underline{\boldsymbol{B}} = \begin{bmatrix} 1 & -1 \\ -2 & -3 \end{bmatrix}, \quad \overline{\boldsymbol{C}} = \underline{\boldsymbol{C}} = \begin{bmatrix} 8.5 & 0 \\ 0 & 8.5 \end{bmatrix}, \quad L_1 = L_2 = 1. \tag{7}$$

In this example, one has $(\|\boldsymbol{B}^*\|_2 + \|\boldsymbol{B}_*\|_2) = 3.884$ and $\|\boldsymbol{Q}\|_2 = 4.131$ and consequently (4) and (5) yield

$$\|\boldsymbol{B}\|_2 \leqslant 3.884 \tag{8}$$

and

$$\|\boldsymbol{B}\|_2 \leqslant 4.131, \tag{9}$$

respectively. Thus, in this example the criterion (4) yields a less conservative estimate of the upper limit of $\|\boldsymbol{B}\|_2$ than the criterion (5). On the other hand, in the example given by [37]

$$\overline{\boldsymbol{A}} = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}, \quad \underline{\boldsymbol{A}} = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix}, \quad \overline{\boldsymbol{B}} = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}, \quad \underline{\boldsymbol{B}} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \quad \overline{\boldsymbol{C}} = \underline{\boldsymbol{C}} = \begin{bmatrix} 4.5 & 0 \\ 0 & 4.5 \end{bmatrix}, \quad L_1 = L_2 = 1, \tag{10}$$