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Systemic risk in multiplex networks with asymmetric coupling and threshold feedback

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HIGHLIGHTS

- We study systemic risk on a two-layer multiplex network with asymmetric coupling strength between layers.
- Systemic risk is underestimated or overestimated by the aggregated representation of a multi-layered system.
- Sharp phase transitions in the cascade size exist depending on the coupling strength.
- We derive mathematical approximations for the phase transitions and we confirm our findings by simulations.

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ABSTRACT

We study cascades on a two-layer multiplex network, with asymmetric feedback that depends on the coupling strength between the layers. Based on an analytical branching process approximation, we calculate the systemic risk measured by the final fraction of failed nodes on a reference layer. The results are compared with the case of a single layer network that is an aggregated representation of the two layers. We find that systemic risk in the two-layer network is smaller than in the aggregated one only if the coupling strength between the two layers is small. Above a critical coupling strength, systemic risk is increased because of the mutual amplification of cascades in the two layers. We even observe sharp phase transitions in the cascade size that are less pronounced on the aggregated layer. Our insights can be applied to a scenario where firms decide whether they want to split their business into a less risky core business and a more risky subsidiary business. In most cases, this may lead to a drastic increase of systemic risk, which is underestimated in an aggregated approach.

1. Introduction

Cascading failures in complex systems can be understood as a process by which the initial failure of a small set of individual components leads to the failure a significant fraction of the system's components. This is due to interconnections between the different components of the system. Such a phenomenon can occur in physical systems such as power grids (e.g. [1–3]), but also in complex organizations like interbank systems (e.g. [4–7]). A general framework to study such cascading failures in networked systems was developed in [8], and extended recently to work in more general topologies in [9].

In many situations, cascading failures can be influenced by the combination of *different types* of interactions between the individual components of the system. This is the case in interbank systems, where banks are exposed to each other via different types of financial obligations (loans, derivative contracts, etc.) (e.g. [10,11]). The bankruptcy of a bank can thus cascade to other banks in non-standard ways. Another example is firms diversifying their activities across different business units, each of which is exposed to cascade risk in its own field of activity.

An important question we wish to investigate in this article is how diversification across different types of interactions can affect the risk of cascading failures. For that purpose, we study the case of a firm that diversifies its activities across a core-business unit and a subsidiary-business unit. Each business unit is exposed to other firms' business units in the same sector of business activity (either core or subsidiary). This means that a business unit can fail (i.e. go bankrupt) as a result of a cascade of failures (i.e. bankruptcies) in the same sector of business activity.

The question of the structuring of a firm into sub-units has been studied from a different angle in the financial economics literature (e.g. [12,13]) and often focuses on the efficiency of the allocation of its resources across different industries. Another question that

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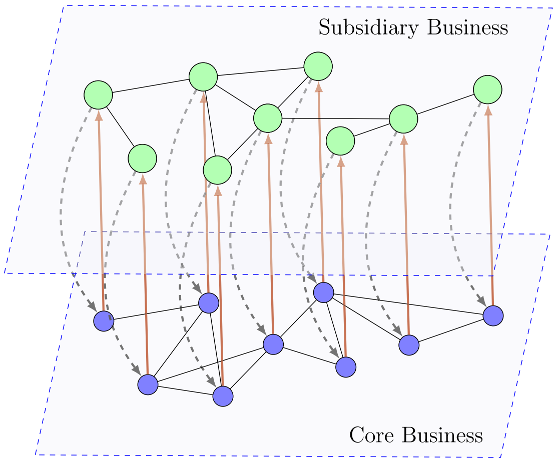

Fig. 1. Illustration of a system with asymmetrically coupled layers. A failure (or bankruptcy) on the Core Business layer implies a failure on the Subsidiary Business layer. This coupling is illustrated by an inter-layer dependency link (red arrow). On the other hand, a failure on the Subsidiary Business layer only decreases a node's failure threshold on the Core Business layer. This coupling is illustrated using dashed black arrows. The intra-layer links represent business relations or other forms of interactions due to normal business. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

has received some attention is that of whether a firm can diversify the risk of its income streams by operating in different business areas. Namely, Levy and Sarnat [14], Smith and Schreiner [15] and Amihud and Lev [16] studied how conglomerates can diversify the risk associated with their revenue streams from the perspective of portfolio theory.

Here, we use a complex networks approach and we view the system of firm activities as an interconnected multi-layered network (see [17–20]). The distinct layers of this network contain individual networks defined by a particular type of interactions according to a given business activity, while the inter-connections between layers allow for cross-layer interactions. In this setting we develop a model where failures (i.e. bankruptcies) on two different network layers affect firms asymmetrically: The first layer represents exposures between the firms in the core business while the second layer represents exposures between firms in the subsidiary business. Failure (i.e. bankruptcy) of a firm's core business unit implies failure of its subsidiary business unit, whereas failure of a firm's subsidiary business unit only causes a shock to the firm's resistance threshold in its core business unit (see Fig. 1 for an illustration). We find that when the coupling strength from the core to the subsidiary layer is varied only slightly, there is a sharp transition between a safe regime, where there is no cascade of failures, and a catastrophic regime, where there is a full cascade of failures. Moreover, when comparing the two-layer network to the single-layer network formed by aggregating the two layers, we find that cascades can be larger on the two-layer network than on the aggregated one. On the other hand, by varying the strength of the feedback between the two layers, we identify the existence of a regime where the two-layer network is safer than the aggregated one and another regime where the reverse holds. This points to the critical importance of the coupling of the layers when structuring a firm into different business units. Also, dealing with aggregated network data that ignores the fine structure of the coupling between different layers can lead to significant underestimation or overestimation of cascade risk.

The article is structured as follows. In Section 2, we describe the two-layer cascade model. In Section 3, we derive a branching process approximation as an approximation for large networks and use it to analyze the aforementioned phenomena. These phenomena are presented in Section 4 where we compare our analytical results with simulations and analyze further the observed phase transitions. In Section 5, we conclude and interpret the consequences of our theoretical investigations for the application to networks of firms that might decide about merging their core and their subsidiary business.

2. Model

We first consider a finite model with N firms. Each firm can be represented by a node i present on each of two different layers: layer 0 (the *core-business* layer) and layer 1 (the *subsidiary-business* layer). Each layer $l \in \{0, 1\}$ has a topology represented by an adjacency matrix G_l. On each layer l, node i can be in one of two states $s^i_l \in \{0, 1\}$, healthy ($s^i_l = 0$) or failed ($s^i_l = 1$). $s^i_0 = 1$ represents the bankruptcy of firm i's core-business unit, whereas $s^i_1 = 1$ represents the bankruptcy of its subsidiary-business unit. This state is determined by two other variables: a node's fragility on a given layer ϕ^i_l, which accumulates the load a node carries, and its threshold θ^i_l on that layer, which determines the amount of load it can carry without failing. Whenever the fragility exceeds the threshold $\phi^i_l \geq \theta^i_l$, the node fails on that layer and cannot recover at a later point in time.

On each layer, we assume that a cascade of failures spreads according to the threshold failure mechanism of Watts [21]. Thus a node fails if a sufficient fraction of its neighbors have failed. The fragility of a node i of degree k^i_l on layer l (i.e. a node with k^i_l neighbors on layer l) can be expressed as

$$\phi^i_l(k^i_l) = \frac{1}{k^i_l} \sum_{j \in \mathrm{nb_l(i)}} s^j_l = \frac{n^i_l}{k^i_l} \tag{1}$$

where $\mathrm{nb_l(i)}$ is the set of nodes in i's neighborhood on layer l and n^i_l is the number of failed neighbors on layer l. This failure mechanism is useful to model a firm diversifying its exposure to failure risk across neighbors: the more neighbors a node has, the less it is exposed to the failure of a single neighbor. A cascade of failures thus starts with an initial fraction of failed nodes. These failures can then spread to their neighbors in discrete time steps. The load ϕ^i_l of a node i is thus updated at each time step t. This model has been studied extensively on single-layer networks, in the context of configuration model type random graphs with a given degree distribution [22–24,7,25], and has been adapted to financial networks of interbank lending [5,6,26]. In [9] a mesoscopic perspective is added by studying conditional failure probabilities given the degree of node. Generalizations of the model to weighted networks can be found in [7,27,9].

In a financial or economic setting, where the nodes are assumed to represent firms that possess simplified versions of balance sheets, the fragility and threshold can be expressed in terms of the liability and capital of a firm. In this case, the fragility ϕ^l_i of a node i represents the loss that a firm encounters divided by its total liability L^i_l in layer l. According to the Watts model, i has the same financial obligation $w^i_l = L^i_l/k^l_i$ to each of its neighbors in layer l, we therefore have $\sum_{j \in \mathrm{nb_l(i)}} w^i_l = L^i_l$ and

$$\phi^i_l(k^i_l) = \frac{\sum_{j \in \mathrm{nb_l(i)}} s^j_l w^i_l}{L^i_l} = \frac{n^i_l}{k^i_l}. \tag{2}$$

The threshold θ^l_i of a node in layer l signifies similarly the ratio between a node's capital buffer C^i_l and its total liabilities L^i_l:

$$\theta^i_l = \frac{C^i_l}{L^i_l}. \tag{3}$$

Consequently, when a node i fails and its fragility exceeds its threshold ($\phi^i_l(k^i_l) \geq \theta^l_i$), equivalently its loss $n^i_l w^i_l$ exceeds its capital