Physica D 313 (2015) 81–98

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

# The effects of wind and nonlinear damping on rogue waves and permanent downshift

C.M. Schober *, M. Strawn

*Department of Mathematics, University of Central Florida, Orlando, FL, USA*

## HIGHLIGHTS

- Rapid changes in the flux and energy are essential for downshift.
- Rogue waves typically do not develop after permanent downshift.
- Linear damping by the wind weakens downshifting.
- Forcing by the wind enhances downshifting.
- Downshift occurs earlier for rogue waves of greater strength.

## ARTICLE INFO

*Article history:*
Received 28 May 2015
Received in revised form
27 August 2015
Accepted 11 September 2015
Available online 21 September 2015
Communicated by J. Bronski

*Keywords:*
Rogue wave
Freak wave
Downshifting
Nonlinear damping
Nonlinear Schrödinger equation
Wind action

## ABSTRACT

In this paper we investigate the effects of wind and nonlinear damping on permanent downshift and the formation of rogue waves in the framework of a HONLS model. Wind effects are incorporated by including a uniform linear damping/forcing term in the model. The strength of the wind, $\Gamma$, is allowed to vary as well as wind duration. Determining permanent downshift is not straightforward and we propose a criteria for permanent downshift related to our numerical experiments.

We consider large ensembles of initial data for modulated unstable Stokes waves with $N = 1, 2, 3$ unstable modes. In the nonlinear damped HONLS evolution we find that permanent downshift is observed whenever the strength of the nonlinear damping $\beta > 0.1$. Notably, rogue waves typically do not develop after the time of permanent downshift, implying that a downshifted sea-state does not allow for any further rogue waves. Incorporating wind effects into the nonlinear damped HONLS model, we find that damping by the wind weakens downshifting while forcing by the wind enhances downshifting.

The proximity of the initial data to unstable plane waves impacts the characteristic features of the rogue waves in the nonlinear damped HONLS evolution. We find that as the initial data is chosen closer to the plane wave, the maximum strength, number, and lifetime of rogue waves increase while the time of permanent downshift decreases. Alternatively, we show that the greater the wave strength, the more rogue waves, or the longer their lifetime, the earlier permanent downshift occurs.



## 1. Introduction

Associated with the Benjamin–Feir or modulational instability, where small perturbations of the Stokes wave lead to exponential growth of the sidebands, is a host of interesting phenomena, two of which are frequency downshifting and rogue waves. One of the simplest descriptions of the modulational instability is provided by the nonlinear Schrödinger (NLS) equation which has been used as a first approximation for modeling rogue waves and addressing downshifting. Lake et al. investigated analytically and experimentally the evolution of the modulational instability [1,2]. In their laboratory experiments, when the initial steepness of the waves was small, the wave trains experienced Fermi–Pasta–Ulam recurrence. On the other hand, when the wave steepness was large, the lower and upper sidebands grew at different rates and the energy was permanently transferred from the carrier to a lower sideband, resulting in permanent frequency downshift. The corresponding numerical experiments using the conservative NLS equation showed a cyclical transfer of energy between the carrier and the sidebands, which is consistent with the observed recurrence for the waves with small steepness. Since the NLS equation preserves the symmetry of the Fourier components, it was not able to describe the downshift dynamics observed for waves with larger steepness.

* Corresponding author.
*E-mail address:* cschober@ucf.edu (C.M. Schober).

http://dx.doi.org/10.1016/j.physd.2015.09.010

The Dysthe equation, obtained by retaining terms through fourth order in an asymptotic expansion for the surface displacement, introduces asymmetry with respect to space translations [3]. The higher order corrections have a significant effect on the evolution of the modulational instability and it was conjectured that the Dysthe equation might capture the permanent downshift observed in the laboratory experiments. However, only temporary downshifting is obtained near the peaks of the modulation in numerical experiments [4]. Even though the momentum is not conserved by the Dysthe equation, there is insufficient change in the flux to allow for permanent downshift. Recently the Dysthe equation was brought into Hamiltonian form, providing a new higher order nonlinear Schrödinger (HONLS) equation [5]. Permanent downshift is not possible in the HONLS equation, even when steep waves are encountered, as the energy and momentum, and thus the spectral center, are conserved.

Although a study of random waves with Gaussian spectrum yielded permanent downshift in an undamped Dysthe's equation [6], the general consensus is that damping plays an important role in downshifting. Recent studies on the effects of dissipation on the Benjamin–Feir instability have shown numerically and in laboratory experiments that linear damping, even when weak, stabilizes the instability [7]. The exponential growth of the sidebands may no longer be saturated by the nonlinearity, rather the growth may be bounded by dissipation before the nonlinear terms become significant [7,8]. However, permanent downshift does not occur in higher order nonlinear Schrödinger models with only linear damping as the energy and momentum decay at the same rate which keeps the spectral center constant. Once nonlinear dissipation was included in Dysthe type models, permanent downshift was obtained for both breaking waves [9] and non-breaking waves [10,11].

The perturbed Stokes wave initial data used in Lake's experiments are close to initial data for special solutions of the NLS equation: the multi-mode homoclinic orbits of unstable Stokes waves with $N$ unstable modes (UMs). We refer to these homoclinic orbits, with $M \leq N$ modes excited, as the $M$ mode spatially periodic breather solutions (SPBs), which include the Akhmediev breathers when $N = 1$. The SPBs capture the nonlinear stage of the modulational instability and are widely used to model rogue waves since they are transient, localized, and steep [12–14]. One and two mode SPBs have analytically been shown to persist in a conservative broadband HONLS equation [13]. In this paper we use modulated unstable Stokes waves initial data to simultaneously numerically investigate downshifting and rogue waves.

In our earlier study on rogue waves and downshifting using the new HONLS equation with nonlinear damping of the mean flow, the initial data were carefully chosen perturbations of unstable Stokes waves with three UMs, close to initial data for steep coalesced three-mode SPB solutions of the NLS equation. Due to the steepness of the waves, permanent downshift (here downshifting of the wavenumber is considered) occurred for all values of the nonlinear damping parameter $\beta$ considered, $0.05 < \beta < 0.75$ in the numerical experiments. Significantly, for Stokes waves with three UMs it was shown that rogue waves did not develop after the wavenumber is permanently downshifted [15].

The nonlinear damping considered is such that steeper waves result in significantly stronger damping (see Fig. 1). A natural question, which we address in this paper, is whether permanent downshift and the cessation of rogue waves after permanent downshift will be obtained when there are fewer unstable modes initially and the waves have smaller strength. Another novel aspect we investigate is how wind affects downshifting and rogue wave formation as well as the relation between the two phenomena. We conduct a broad numerical investigation of the effects of wind and nonlinear damping on permanent downshift and the formation of rogue waves in the framework of the HONLS model. Wind effects are incorporated by including a uniform linear damping/forcing term in the model. The strength of the wind, $\Gamma$, is allowed to vary as well as the duration of the wind, i.e. the wind acts for limited time $0 \leq t \leq T_{wind}$. Additional questions addressed in the current study are: (i) whether the proximity of initial data to underlying instabilities of the plane wave affects rogue waves and downshifting and how to measure this and (ii) how the strength of the waves affects the time of permanent downshift.

We consider large ensembles of initial data for modulated unstable Stokes waves with $N$ UMs (referred to as the $N$ UM regime). The initial data used in these experiments are more general than in the earlier study since the initial data are generic perturbations of Stokes waves which yield waves with smaller strength. Typically we examine the $N = 1, 2$ UM regime. For some of the issues not previously addressed, e.g. the combined effects of wind and nonlinear damping, the $N = 3$ UM regime is also examined. We assume the waves do not break and that asymmetry develops on the same timescale as the action of the wind and nonlinear damping.

Section 2 provides background material: the properties of the governing equations, a linear stability analysis of Stokes waves for the new nonlinear damped HONLS, and the setup of the numerical experiments. As a point of comparison for the nonlinear damped HONLS evolution, we review the construction of rogue waves in the NLS equation and their appearance in the conservative HONLS equation (Section 3). Linear damping by the wind is examined in Section 4 which shows only temporary downshifting, as the theory predicts. In Section 5 nonlinear damping is introduced to the model and its effect on rogue waves and downshifting is quantified. Permanent downshift is observed, even for waves with smaller steepness, whenever the strength of the nonlinear damping $\beta > 0.1$. Notably, rogue waves typically do not develop after the time of permanent downshift, implying that a downshifted seastate does not allow for any further rogue waves. Additionally, we examine how the proximity of initial data to the plane wave shown impacts downshifting and the features of the rogue waves. The effect of wind and nonlinear damping on rogue waves and downshifting is considered in Section 6 where it is shown that damping by the wind weakens downshifting while forcing by the wind enhances downshifting.

## 2. Analytical background

### 2.1. Governing equations

Recently Gramstad and Trulsen brought the Dysthe equation, which approximately describes the evolution of slowly modulated periodic wave trains in deep water, into Hamiltonian form [5]. We add a linear damping/forcing term to the model to incorporate wind effects and a nonlinear damping term, the $\beta$-term, to model damping of the mean flow:

$$iu_t + u_{xx} + 2|u|^2u + i\Gamma u + i\epsilon\left(\frac{1}{2}u_{3x} - 8|u|^2u_x - 2ui(1+i\beta)\left[\mathcal{H}\left(|u|^2\right)\right]_x\right) = 0, \quad (1)$$

where $u(x, t)$ is the complex envelope of a wavetrain, $\epsilon \geq 0$ and $\mathcal{H}\left(f(x)\right) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(\xi)}{x-\xi}d\xi$ is the Hilbert transform of $f$. Periodic boundary conditions, $u(x, t) = u(x + L, t)$, are considered. When $\beta = \Gamma = 0$ Eq. (1) is referred to as the HONLS equation.

As can be deduced from Eq. (3), nonlinear damping requires $\beta > 0$. Fig. 1 shows $|u(x, t)|$ (solid line), and the corresponding $\beta$-term $|u\left[\mathcal{H}\left(|u|^2\right)\right]_x|$ (dashed line), when the wave train is strongly modulated at $t = t^*$. The $\beta$ term is large only near the two steep crests at $x \approx -7$ and $x \approx 2.5$ while the $\beta$ term is nearly negligible