Journal of Food Engineering 127 (2014) 85–92

Contents lists available at ScienceDirect

Journal of Food Engineering

journal homepage: www.elsevier.com/locate/jfoodeng

Freezing time formulas for foods with low moisture content, low freezing point and for cryogenic freezing

Q. Tuan Pham *

School of Chemical Engineering, University of New South Wales, Sydney 2052, Australia

A R T I C L E   I N F O

Article history:
Received 3 October 2013
Received in revised form 21 November 2013
Accepted 6 December 2013
Available online 12 December 2013

Keywords:
Food freezing
Freezing time formula
Biot number
Cryogenic freezing

A B S T R A C T

Existing food freezing time formulas have not been tested on low moisture, low freezing point foods or cryogenic temperatures. A systematic numerical experiment was therefore performed to generate freezing times over a wide range of parameters, never before covered in the literature, yet still within industrial practice. The results were used to evaluate four well-known freezing time formulas. Pham's first (three-stage) method has the best theoretical basis and fewest empirical parameters, and agrees best with numerical predictions. By applying simple correction factors, this method agrees with numerical predictions to within ±10% while Pham's second (two-stage) method is almost as accurate. For the freezing time of fresh foods ($T_f \geqslant -1.5\,°C$), Pham's first method can be used without modification. The corrected formulas remain accurate under cryogenic freezing conditions.

1. Introduction

Freezing is a widely used food preservation method, and the calculation of freezing time is an essential step in the design and optimisation of refrigeration plants, as well as in ensuring the maintenance of food quality. However, this calculation is made difficult by the complex composition and behaviour of biological materials. Unlike pure water, foods do not solidify at a sharp temperature. As ice forms it separates out of the solutes in the food, causing the remaining liquid solution to become more concentrated and the freezing point to fall. Latent heat is therefore released gradually over a range of temperature. Furthermore, some of the moisture may be "bound" and not freeze at all, at least at the temperatures encountered in industrial food freezing.

The freezing time of foods can be calculated by any of the following classes of methods: analytical formulas, empirical formulas or numerical methods. Numerical methods such as the finite difference method (FDM) or finite element method (FEM) are rigorous and precise. However, not all food technologists have the skills to write a numerical program or have access to commercial software. An even more serious drawback is that to take full advantage of a numerical program, the functions relating food thermal properties such as thermal conductivity, density and enthalpy to temperature must be available, which is rarely the case. Although predictive methods and programs for food properties are available, such as COSTHERM (Miles et al., 1983) or the method used later in this paper, they are still subject to errors due to the assumptions made. Therefore, other approaches are still widely used. These simplified methods usually need only the values of food thermal properties at one or two specific temperatures.

Exact analytical solutions are available for some idealised cases which never happen in practice. They are useless for practical engineers but are often used as the starting point to develop more useful prediction methods. Thus, Plank's (1913a,b) equation, which assumes zero sensible heat and a sharp freezing point, has been used to derive several widely used approximate formulas such as Cleland and Earle's (1977, 1984), Mascheroni and Calvelo's (1982), Pham's (1984, 1986), Ilicali and Saglam (1987), Lacroix and Castaigne (1988), Salvadori and Mascheroni's (1991, 1996) and Salvadori (1994).

The best known empirical methods have been validated against a relatively large body of experimental data and this is the main justification for their use. Some of the methods have been more thoroughly tested than others. Among the most systematic comparative tests are those of Cleland (1990), Pham and Willix (1990) and Becker and Fricke (1999), using some of the following databases: Cleland and Earle (1977, 1979a,b), Hung and Thomson (1983), de Michelis and Calvelo (1982), Ilicali and Saglam (1987), Pham and Willix (1990) and Tocci and Mascheroni (1994).

However, there are serious gaps in the datasets available. Firstly, they were mostly obtained with high moisture content foods, about 70% water or more (meat, fish, fruit). Secondly, the majority of experiments were not even done on food, but on methylcellulose gel (Tylose), a food analogue with supposedly similar properties to meat. Thirdly, due to the cost in time and money of performing experiments, not all combinations of parameters could be covered so there are lots of gaps in the data. There are few tests

* Tel.: +61 2 9385 5267; fax: +61 2 9385 5966.
E-mail address: tuan.pham@unsw.edu.au

0260-8774/$ - see front matter
http://dx.doi.org/10.1016/j.jfoodeng.2013.12.007

Nomenclature

Acronyms

CE	Cleland and Earle's freezing time prediction method
FD	finite difference method
P1	Pham's first (1984) freezing time prediction method
P2	Pham's second (1986) freezing time prediction method
PL	Plank's freezing time equation
SM	Salvadori and Mascheroni freezing time prediction method

Symbols

Bi	Biot number based on frozen food, hR/k_f
Bi_u	Biot number based on unfrozen food, hR/k_u
c	specific heat (may or may not include latent heat of phase change, depending on context), J kg^{-1} K^{-1}
e	relative error, $t_{Formula}/t_{FD} - 1$
h	surface heat transfer coefficient, W m^{-2} K^{-1}
H	enthalpy, J kg^{-1}
ΔH_{10}	enthalpy change of the product between T_f and −10 °C, J kg^{-1}
$\Delta H_{\mathrm{Tf}-10}$	enthalpy change of the product between T_f and $T_f - 10$, J kg^{-1}
k	thermal conductivity, W m^{-1} K^{-1}
L	latent heat of freezing per unit mass of food, J kg^{-1}
Pk	Plank number, $c_u(T_i - T_f)/L$
Pk'	Plank number (Cleland and Earle's version), $c_u(T_i - T_f)/\Delta H_{Tf-10}$
R	half-thickness of slab, m
R_T	ratio of freezing point depression to air temperature in °C, $(T_0 - T_f)/(T_0 - T_a)$
Ste	Stefan number, $c_f(T_f - T_a)/L$
Ste'	Stefan number (Cleland and Earle's version), $c_f(T_f - T_a)/\Delta H_{Tf-10}$
t	freezing time to a centre temperature of T_c, s (subscript denotes calculation method)
T_0	freezing point of pure water (0 °C)
T_a	environment temperature, °C
T_c	final centre temperature, °C
T_f	initial freezing point, °C
T_i	initial food temperature, °C
x_b	bound water mass fraction
x_c	carbohydrate mass fraction
x_f	fat mass fraction
x_i	ice mass fraction
x_m	mineral mass fraction
x_p	protein mass fraction
x_w	total water mass fraction
y	space coordinate, m
α	thermal diffusivity $k/\rho c$, m^2 s^{-1}
ρ	density, kg m^{-3}

Subscripts

f	frozen food
u	unfrozen food

at very high Biot number, and practically no tests for low moisture foods or those with low freezing point (below about −1.5 °C) due to added salts, such as salted butter or ham. Fourthly, there has been no systematic experiments in the cryogenic temperature range.

Numerical methods constitute a promising way to generate "data", but as mentioned above their accuracy depends on accurate knowledge of thermal properties and how they vary with temperature. Fortunately, in recent years, enough progress has been made to allow the prediction of these property–temperature relationships with some confidence.

This work therefore aims to numerically generate freezing time data for a one-dimensional geometry (slab), to use this data to verify some well-known approximate formulas over a wider range of parameters than they were designed for, and to suggest correction factors to extend their range of usefulness, particularly for foods with low moisture contents and low freezing points such as butter and ham. A second objective is to see if these methods are applicable to cryogenic freezing.

2. Theory

This work is purely concerned with freezing in one dimension, i.e., the freezing of slabs of finite thickness and infinite area. For other shapes the freezing time can be calculated from that of a slab by multiplying it by a suitable shape factor (Hossain et al., 1992a,b,c; Cleland et al., 1987a,b). The governing equation for this situation is Fourier's transient heat conduction equation:

$$\rho \frac{\partial H}{\partial t} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) \tag{1}$$

where the enthalpy H consists of a sensible heat component and a latent heat component. Assuming that all the latent heat L is released at a single temperature T_f and that the sensible heat component is zero, Plank (1913a,b) derived the equation for freezing time.

$$t_{PL} = \frac{\rho L}{T_f - T_a}\left(\frac{R}{h} + \frac{R^2}{2k_f}\right) \tag{2}$$

which can also be written as (Pham, 1986).

$$t_{PL} = \frac{\rho L R}{h(T_f - T_a)}\left(1 + \frac{Bi}{2}\right) \tag{3}$$

where $Bi = hR/k_f$ is the Biot number based on frozen food thermal conductivity. Due to its unrealistic assumptions Plank's equation grossly underpredicts real-life food freezing times. Several attempts have been made to improve it by adding terms or factors to take into account the sensible heat and/or the gradual release of latent heat. We will only look at a few of the best known.

Mascheroni and Calvelo (1982) divide the total freezing time into precooling, phase change and subcooling (or postcooling) periods. The phase change time is calculated from Plank's equation (Eq. (3)), while the precooling and subcooling times are calculated from analytical expressions (Carslaw and Jaeger, 1959) which assume uniform initial temperature and constant thermal properties for each period:

$$t = t_{precool} + t_{PL} + t_{subcool} \tag{4}$$

Because the precooling and subcooling time expressions involve infinite series, a computer program or a graphical solution is necessary for those periods. For this reason the method is not widely used, but its methodology of adding cooling and phase change times forms the basis for other methods such as those of Pham (1984, 1986), Ilicali and Saglam (1987) and Lacroix and Castaigne (1988).