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Asymmetry of the stress tenor in granular materials

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ABSTRACT

One of the basic assumptions of the micropolar theory is that the stress tensor is not symmetric. In this paper, asymmetry of the stress tensor is studied with discrete element method and averaging method. The change of the skew symmetric part of an asymmetric tensor with the rotation of the coordinate system is shown graphically. Averaging method is used to obtain stress tensor from a DEM simulation of biaxial test. Stress asymmetries at different locations, scales and time steps are studied. The importance of the asymmetric stress for setting up a constitutive model for granular materials is discussed.

1. Introduction

In classical continuum mechanics, the Cauchy stress tensor used to describe the stress state of materials is assumed to be symmetric. This holds for a large class of materials and cases in continuum modeling [8,9,16,31,45,46,48]. However, the stress tensor is not necessarily symmetric. The asymmetry of stress becomes significant if the microscopic scale material behavior is considered. This has been proved by several researches [2,3,10,17,23]. In some advanced continuum theories, such as micropolar continuum theory [13,22,25,35,36,42], asymmetric stress tensors are used. The balance of the skew symmetric part requires additional degrees of freedom and characteristic length. For granular materials, especially in strain localized regions [32–34,37,38,44,47,49] and boundaries [43], the asymmetry stresses are shown to have important effects. Although the symmetric parts of stress tensor are important, neglecting the asymmetric part can lead to inaccuracy for many microscopic problems [1,12,29,39]. Hence, this paper will focus on the asymmetric parts of stress tensors.

In this paper, we first start with the analysis of a general asymmetric tensor, which can be decomposed into a symmetric part and a skew symmetric part. The change of the skew symmetric part with coordinate is shown graphically in the same way that shear stresses are shown in Mohr circles.

A simulation of biaxial test is carried out using discrete element method (DEM). The sample is loaded until a shear band can be observed. Stresses in this DEM model is studied using the stress homogenization/averaging method, which is able to obtain stress tensor from discrete elements. The stress tensors depend on the size of the averaging volumes. Hence, several different averaging sizes are used to obtain stress inside and outside of the shear band. Since the fabric of granular material in the DEM calculations varies over time, the fluctuations of stress tensors are also shown for different cases. Finally, the significance of stress asymmetry for setting up a micropolar constitutive model is discussed.

2. Asymmetric tensor

In this section, we consider an arbitrary asymmetric stress tensor, which is a second order tensor with nine independent components. An asymmetric tensor can be expressed as sum of a symmetric and a skew symmetric tensor. The stress state of a symmetric stress tensor can be shown graphically by Mohr circles. In this paper, a similar way is used to show the skew symmetric stress graphically. The following steps need to be followed to plot these graphics.

First, the tensor is transformed to a certain coordinate system, in which only the principle stresses and the skew symmetric stresses exist, i.e., no shear stresses.

$$\begin{pmatrix} \sigma_1 & s_3 & -s_2 \\ -s_3 & \sigma_2 & s_1 \\ s_2 & -s_1 & \sigma_3 \end{pmatrix} \tag{1}$$

where $\sigma_1, \sigma_2, \sigma_3$ are principle stresses and s_1, s_2, s_3 are skew symmetric stresses. Starting from this matrix, we rotate the coordinate system with the principle axis to show the change of different terms. If the

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coordinate system rotates around the z axis for an angle α, the matrix becomes:

$$\begin{pmatrix} \mathbf{C} & -\mathbf{S} & 0 \\ \mathbf{S} & \mathbf{C} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 & s_3 & -s_2 \\ -s_3 & \sigma_2 & s_1 \\ s_2 & -s_1 & \sigma_3 \end{pmatrix} \begin{pmatrix} \mathbf{C} & \mathbf{S} & 0 \\ -\mathbf{S} & \mathbf{C} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} \sigma_1\mathbf{C}^2+\sigma_2\mathbf{S}^2 & s_3+(\sigma_1-\sigma_2)\mathbf{CS} & -s_2\mathbf{C}-s_1\mathbf{S} \\ -s_3+(\sigma_1-\sigma_2)\mathbf{CS} & \sigma_2\mathbf{C}^2+\sigma_1\mathbf{S}^2 & s_1\mathbf{C}a-s_2\mathbf{S} \\ s_2\mathbf{C}+s_1\mathbf{S} & -s_1\mathbf{C}+s_2\mathbf{S} & \sigma_3 \end{pmatrix}$$
$$= \begin{pmatrix} \sigma_1\mathbf{C}^2+\sigma_2\mathbf{S}^2 & (\sigma_1-\sigma_2)\mathbf{CS} & 0 \\ (\sigma_1-\sigma_2)\mathbf{CS} & \sigma_2\mathbf{C}^2+\sigma_1\mathbf{S}^2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} + \begin{pmatrix} 0 & s_3 & -s_2\mathbf{C}-s_1\mathbf{S} \\ -s_3 & 0 & s_1\mathbf{C}-s_2\mathbf{S} \\ s_2\mathbf{C}+s_1\mathbf{S} & -s_1\mathbf{C}+s_2\mathbf{S} & 0 \end{pmatrix}$$

where C is cosα and S is sinα.

Hence, the skew symmetric term s_1 and s_2 change to $s_1\mathbf{C}_\alpha - s_2\mathbf{S}_\alpha$ and $s_2\mathbf{C}_\alpha + s_1\mathbf{S}_\alpha$. For two angles α_1 and α_2 with the following relationship:

$$\alpha_2 = \alpha_1 + \frac{\pi}{2}. \tag{2}$$

We have,

$$\mathbf{C}_{\alpha_1} = \mathbf{S}_{\alpha_2} \tag{3}$$

$$-\mathbf{S}_{\alpha_1} = \mathbf{C}_{\alpha_2} \tag{4}$$

and

$$s_1\mathbf{C}_{\alpha_1} - s_2\mathbf{S}_{\alpha_1} = s_2\mathbf{C}_{\alpha_2} + s_1\mathbf{S}_{\alpha_2}. \tag{5}$$

Hence, the difference between s_1 and s_2 is an angle of $\pi/2$. If we plot the s_1 versus σ_2 and s_2 versus σ_1, the two curves will follow the same trace. Hence, only one curve is needed to represent the change of s_1 and s_2 with the rotation of z axis. Similar relationships also hold for coordinate rotation around x or y axises. Hence, for an arbitrary asymmetric matrix, we can plot the change of skew symmetric tensor versus coordinate system as shown in Fig. 1.

It can be seen that the skew symmetric parts are shown with three butter fly shaped plots. The η axis is the skew symmetric part in one direction, and the ξ axis is the sum of normal stresses in two perpendicular directions. In this way, we can put the three curves together. The left and right boundaries of the three curves touch each other and the corresponding ξ coordinate equals to the sum of two principle stresses. The plots indicated in red, green and blue in Fig. 1 represent skew symmetric stress perpendicular to 1, 2 and 3 directions. We can plot these curves for any asymmetric tensor and this relationship always hold. See Table 1 for more plots of asymmetric tensors.

The reason of using the sum of two normal stresses as ξ axis is explained in the following. Consider the rotation of one point in a material (Fig. 2), the skew symmetric stress generates a moment around the rotational direction. The two normal stresses are both perpendicular to the direction of rotation and have the same effect on the rotation. Hence, the sum of these two normal stresses should be used. The relationship between the skew symmetric stress and the sum of normal stresses is similar to the relationship between shear stress and normal stress in the Mohr circle. Hence, the butter fly plots use the skew symmetric stress as the η axis and the sum of two other normal stresses as the ξ axis.

From the butterfly shaped plots, it is easy to find out where each asymmetric stress reaches maximum and minimum value, and what are the corresponding normal stresses.

However, unlike the Mohr's circle, with which yield surfaces for shearing can be defined, the butter fly plots of asymmetric tensor alone cannot define any yield surface for rotation. The reason is that rotation of material is a microscopic scale material behavior which depends highly on the scale, while stress in continuum mechanics is independent of the size. Hence, the asymmetry of the stress tensor cannot define any yield conditions for the rotational degrees of freedom. Scale dependent terms such as couple stresses will be needed. It remains an open question how these plots can be further used. Nevertheless, the butter fly plots offer a graphical tool to show the change of stress asymmetry with coordinate systems. These plots are later used to show the averaged stress of DEM simulations.

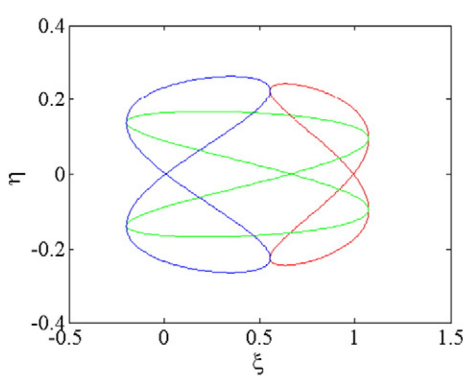

Fig. 1. Plot of skew symmetric term according to the sum of normal stresses in two other directions.

3. Stress asymmetry in granular material

In granular materials, the contact force networks [27,41], define the stress tensor, which is generally non-symmetric. According to Bardet and Vardoulakis [2], the asymmetry of stress tensor can be represented with:

$$e_{ijk}\overline{\sigma}_{jk} = \frac{1}{V}\sum_{c\in I} e_{ijk}\left(x_j^b - x_j^a\right)f_k^c = -\frac{1}{V}\sum_{e\in E} M_i^{ae} = \frac{1}{V}\sum_{e\in E} e_{ijk}x_k^{ae}f_k^e \tag{6}$$

$$\overline{\sigma}_{ij} - \overline{\sigma}_{ji} = \frac{1}{V}\sum_{e\in E}\left(x_i^{ae}f_j^e - x_j^{ae}f_i^e\right) = -\left(e_{ijk}-e_{jik}\right)\frac{1}{V}\sum_{e\in E} M_k^{ae} \tag{7}$$

where $\overline{\sigma}_{ij}$ is the average stress, e_{ijk} is the permutation tensor, V is the average volume. I and E are groups of internal and external contact points. x_j^a, and x_j^b denote the coordinates of the centers of particles a and b in direct contact at internal contact point $c\in I$. At this interparticle contact point the interparticle force $f_k^c = f_k^{ac} = -f_k^{bc}$ is applied. x_i^{ae} stands for the coordinates of the center of particle a, on the surface of which an external force f_j^e occurs at external contact point $e\in E$. M_i^{ae} denotes the external moment about the center of this particle at the external contact point $e\in E$, namely (see Eq. (21) of [2])

$$M_i^{ae} = e_{ijk}\left(x_j^e - x_j^{ae}\right)f_k^e + m_i^e \tag{8}$$

in which the external actions f_k^e and m_i^e represent the external contact force and contact moment respectively.

Bardet and Vardoulakis [2] conclude that there is stress asymmetry in granular media even when the particle contacts do not transmit moments. The amplitude of stress asymmetry decreases with the ratio V/S between surface S and volume V. Here, we make use of the averaging method to verify this relationship.

4. DEM and averaging method

It has been shown by previous studies that the material behavior of granular materials can be studied with discrete element methods [14,15,30,40,41]. In order to study the micro–macro relationships in granular materials, homogenization/averaging methods are used [7,19, 21,24,28].

A biaxial test is simulated with DEM using the commercial software PFC (particle flow code) 3D 3.1. The coordinate directions are stipulated