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Interplay between the inclusions of different sizes and their proximity to the wall boundaries on the nature of their stress distribution within the inclusions inside particulate packing

S. Joseph Antony [a],*, David Chapman [a], S. Judes Sujatha [b], Thabit Barakat [c]

[a] Institute of Particle Science and Engineering, School of Chemical and Process Engineering, University of Leeds, LS2 9JT, UK
[b] Department of Civil Engineering, University College of Engineering, Nagercoil 629004, India
[c] Department of Physics and Astronomy, College of Science, King Saud University, PO Box 2455, Riyadh, Saudi Arabia

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ABSTRACT

Micromechanical responses of granular materials are complex to understand when their behaviour is viewed in a single grain-scale. Experimental sensing of stresses within a grain-scale inside three dimensional particulate packing is still difficult to perform. In this work, photo stress analysis tomography (PSAT) is used to sense the fundamental nature of the stress experienced by different sizes of optically-sensitive inclusions inside granular packing (quasi-three dimensional) under an external axial-compression loading. The distribution of the maximum shear stress and the direction of the major principal stress experienced by the inclusions are analysed to understand the interplay between the size of the inclusions and their proximity to the wall boundary. The outcomes of this study provide a new understanding on the dual nature of stress transmission experienced by the inclusions, as a result of the combined size and wall effects. Relatively large inclusions experience dominantly shear stress close to the wall boundaries while this nature tends towards hydrostatic away from the wall boundaries. Smaller size inclusions could experience shear at both close to and away from wall boundaries of the granular assembly. Computer simulations using three-dimensional discrete element method (DEM) are also performed to compare the qualitative nature of stress experienced by inclusions inside particulate media. Qualitatively, the simulation results also agree with the experimental outcomes, that an increase in the relative size of the inclusion decreases its ability to experience shear. Using DEM simulations, the fabric structure of the inclusions is examined in depth under mechanical loading. An increase in the size of the inclusions tends to decrease the fabric anisotropy of the contacts and in particular the strong contacts, surrounding them. Hence the microscale origin of the weak mobilisation of shear in the large inclusions could be attributed to their relatively weak fabric anisotropy of the strong contacts surrounding them. These findings help to advance our understanding of the micromechanics of particulate systems, due to their size and proximity to wall boundaries: different sizes of the particles could sustain different nature of stresses within single-particle scale depending on their proximity to the wall boundaries.

1. Introduction

Worldwide industries process a large proportion of feed stocks in granular and powder form. These include manufacturing ingredients of pharmaceutical tablets, food, cements, ceramics, chemicals and energy materials. Micromechanical behaviour of granular materials, which is generally attributed to enlightening single-grain scale characteristics in granular assemblies and their links to bulk scale properties if any, is complex to understand [1]. Generally they differ from that of conventional states of matter viz., solid, liquid and gases, and often they possess their combined properties [1–3]. For example, a number of studies have focused on understanding the force transmission characteristics of granular materials under external loading environments [1–5]. It is widely recognised that the force-transmission inside granular assemblies occurs via inter-particle contacts in a non-homogeneous manner through a chain-like network of contacts, often referred to as force chains [2–5]. Such observations have been made from photo elastic studies of birefringent grains [6–9], computer simulations using discrete element method (DEM) [10] and combined FEM-DEM [11]. For the granular materials, photo elastic studies were mostly reported for the two dimensional conditions whereas the DEM simulations accounted for both the two dimensional and three dimensional conditions [4, 10–13]. The link between the signature (structural alignment) of the force network and bulk mechanical properties of granular systems is quite significant [14]. For example, studies have attributed the microscopic origin of the bulk shear strength of granular materials, to the fabric alignment of the strong-force transmitting contacts under different

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* Corresponding author.
E-mail address: S.J.Antony@leeds.ac.uk (S.J. Antony).

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external loading conditions [5,15,16]. Strong contacts account for a small proportion (about 25–30%) of all the contacts [2–5]. The strong contacts form the solid backbone to mobilise the shear strength, whereas the particles that share the weak contacts mostly account for the hydrostatic pressure (liquid-like behaviour) [2–5]. Similar to the force network, studies have also identified displacement networks in granular assemblies which account for their dilation and energy distribution characteristics [4].

In this work, we focus on the stress-transmission properties of inclusions in granular assemblies, with the emphasis being on the effects of their size and proximity to the wall boundaries when subjected to axial compressive loading. It is widely recognised that the size effects of particles could play a significant role on the compaction properties of granular assemblies [17]. In this case, size refers to the size distribution of the particles [18], relative size of particulate inclusions to their surrounding particles [19,20] and sample size which in turn depends on the wall dimensions and the condition of the compressing chamber [21]. What is still lacking is clear information on how individual particles of different sizes, especially relatively big particles, distribute shear stress inside granular packing close to, and away from, the wall boundaries under external loading environments. This understanding is important, for example, in designing the size of functional active ingredients of pharmaceutical tablets [18,22] and particulate composites [23]. This is also relevant for modelling particles more accurately in the DEM simulations. In the past, DEM simulations have been used for understanding the inter-particle interactions, the internal and macroscopic properties of granular assemblies under mechanical loading. However, they have not yet advanced to a level where the stress distribution characteristics *within* individual particles inside granular packing can be analysed easily. Conventional DEM simulations do not rigorously account for how stresses are distributed within individual particles [10,20,21]. In some FEM based simulation studies, individual grains were discretised into a large number of elements to predict stress distribution characteristics within the particles [1]. However due to the massive level of computing resources required to account for a large number of such particles in granular assemblies, using this methodology, information on the load transfer characteristics within single-particle scale inside granular assemblies is still not well established. Photo stress analysis tomography (PSAT) [8] has been applied to probe the stress distribution characteristics of inclusions in different surroundings. In these studies, a known size of two dimensional inclusions (circular disk) was surrounded by three dimensional grains in such a way that the thickness of the inclusion and the grains were the same (quasi-3D experiments). Studies were performed using a number of positions for a single-size inclusion from the wall boundaries [8], or different sizes of inclusion positioned far away from walls [9] to sense the shear stress distribution experienced by the inclusion inside the granular assemblies under mechanical loading. They help to advance understandings on the stress distribution characteristics within single-particle scale. This aspect is generally difficult and computationally intensive, though not impossible to study using DEM simulations [10], especially when the size of the inclusion is much greater than the size of surrounding particles. In DEM, average stress calculations are based on inter-particle contact forces and, variations of stresses within particles, are not rigorously accounted for the stress calculations [10]. In the present work, we use PSAT methodology [8,9] to obtain insights on the interplay between the size effects of the inclusions and their proximity to wall boundaries, with an emphasis on the nature of stress distribution experienced by them inside granular assemblies subjected to axial compression.

2. Experiments

2.1. Methodology

In the present study, experiments were conducted using PSAT [8] to measure the maximum shear stress and the direction of principal stresses at any points of interrogation on the birefringent inclusions. Detailed explanations on the working principles of photo stress analysis methodology for granular assemblies can be found elsewhere [24,25], however briefly stated here. Basically, birefringent particles optically respond to stress when viewed under a circular polariscope setup [9,24,25]. Depending on the level of the induced stress, they display contours of fringes of different orders depending on the retardation of light passing through the fast and slow optical axes at the point of interest. Using the stress-optic law [25] the order of the fringes (retardation) can be related to the magnitude of the maximum shear stress τmax ($=(\sigma_{11}-\sigma_{33})/2$, where σ_{11} and σ_{33} are the major and minor principal stress respectively). Also the direction of the principal stresses can be obtained anywhere on the birefringent inclusion by using a plane polariscope setup [25].

2.2. Experimental setup

Fig. 1 shows the axial compression chamber made of glass in which a birefringent polymer disk (circular) inclusion with Young's Modulus 2.9 MPa and Poisson's Ratio 0.4 was embedded in the middle of the particulate bed comprising of a uniformly sized 4 mm diameter cohesionless and Vigna Radiata rigid beads (spheres) in random packing [8]. Initial packing density is 0.73 g/cm^3. The samples were prepared identically in all tests reported here, and follow a uniform protocol (layered filling of grains in random packing) and hence the tests repeatable [8]. This is verified by conducting the experiments three times for a given inclusion size. The nature of the stress distribution characteristics presented later in this paper was consistent in the multiple experiments for each case of the inclusion. The size ratio (SR) of the inclusion, which is the ratio between the diameters of the inclusion and the surrounding beads, was varied as 2.5, 4 and 7. The minimum size of the inclusion was kept to at least twice the size of the surrounding beads to ensure that the embedded inclusion was not trapped between the weak force chains (dead zones at micro scale) [2,4,5]. The thickness of the inclusion disk is 4 mm, which is also equal to the diameter of the surrounding matrix beads (quasi-3D condition). The inter-particle friction between the beads and beads-inclusion is 0.3. The wall boundaries were rigid and kept smooth as much as possible. The friction coefficient between the beads and wall material is about 0.006. The applied loading P pertains to quasi-static, uni-axial compression across the width of the bed (Fig. 1).

To understand the combined effects of the size ratio of inclusions and wall effects, experiments were performed using different positions of inclusion from the wall boundaries represented in terms of λd where λ varies between about 0–5 and d is the diameter of the inclusion (Fig. 1). The inclusion is positioned at a fixed height from the loading pad. More details of the experimental compaction rig and protocol of testing used here can be found elsewhere [8]. Birefringent images for the distribution of maximum shear stress within sensor particles (inclusions) were recorded and analysed, both for the magnitude of maximum shear stress distribution and direction of the major principal stress within the sensor particles for different cases of the size ratio, loading level and position of the inclusion from the wall boundary within the granular bed.

3. Results and discussions

Profiles of the maximum shear stress distribution for different sizes of the inclusions in contact with the wall ($\lambda = 0$) for typical cases of compression load level (P) is presented in Fig. 2 below. The direction of the major principal stress distribution is also overlaid on the images. It is evident that the maximum shear stress distribution occurs in a non-homogeneous manner within the inclusions in contact with the wall. The extent of this non-homogeneity depends on the size of the particles. In the case of the lowest size ratio of the inclusion (size ratio 2.5), the direction of the major principal stress within the inclusion mostly acts