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Similitude in cyclone separators

J.S. Ontko

National Energy Technology Laboratory, U.S. Department of Energy, 3610 Collins Ferry Road, Morgantown, WV 26505, USA

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ABSTRACT

Criteria for similitude in reverse flow cyclone separators are developed in this paper explicitly including the inlet particulate probability distribution. The application of these criteria is demonstrated by example using data from the literature. Some practical points to consider when using cyclone similarity relations are presented in the Conclusion.

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1. Introduction

The reverse flow cyclone separator, shown schematically in Fig. 1, provides a simple and robust method of removing particles from a fluid transporting them. It captures a fraction of the incoming particulate mass, $\overline{\eta}$, and allows the remainder, $\overline{v}$, to escape; $\overline{\eta}+\overline{v}=1$. It also segregates the inlet mass cumulative probability distribution, Ω, into captured and escaped distributions, denoted G and H respectively. The fractional or grade efficiency, η, partitions Ω into G and H, all of which may be functions of particle size and other variables. If (1) $0 \le \eta \le 1$ and (2) η is continuous,

$$\overline{\eta}=\int_0^1 \eta \; d\Omega$$
$$G(\Omega^*)=\frac{1}{\overline{\eta}}\int_0^{\Omega^*} \eta \; d\Omega$$
$$H(\Omega^*)=\frac{1}{\overline{v}}\left[\Omega^*-\int_0^{\Omega^*} \eta \; d\Omega\right]$$

$0 \le \Omega^* \le 1$. These equations can account for any number of random variates, which may be continuous or discrete, in the absence of attrition or agglomeration [1]. To estimate fractional efficiency from data, partition the domain of Ω into disjoint subsets such that the union of the subsets is equal to the domain. Let $\Delta\Omega_i$ and ΔG_i denote the respective inlet and captured probabilities associated with subset i. The fractional efficiency in the ith subset is estimated by $\hat{\eta}_i=\overline{\eta}\Delta G_i/\Delta\Omega_i$, if $\Delta\Omega_i \neq 0$. A static pressure drop, Δp, is developed across the cyclone when in operation.

E-mail address: john.ontko@netl.doe.gov.

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2. Theory

We restrict our attention to a steady, incompressible, isothermal, and chemically unreactive process in which gravitational and electrostatic effects as well as particle attrition may be neglected. Consideration of the characteristic variables (listed in the Nomenclature) influencing the fluid and particle mechanics in a cyclone under these restrictions leads to

$$f\left(\rho_f, V, D, \mu, \varepsilon, \lambda_1, \ldots, \lambda_j, \eta, \Omega, \overline{\eta}, \Delta p\right)=0.$$

where f is an unknown function which is assumed to be dimensionally homogeneous, dimensionally invariant and in which the numerical values of the arguments are positive real numbers. It is understood that η and Ω stand here for the variables and parameters required to completely specify both functions. We may apply the Π theorem [2] to establish the conditions required for similitude, which yields

$$F(N_{Re}, \varepsilon, \Lambda_1, \ldots, \Lambda_j, \eta, \Omega, \overline{\eta}, N_{\Delta p})=0$$

where F is another unknown function. Now consider two cyclones, one of which is denoted the model and the other the prototype. If the design ratios in the model, $\Lambda_{i,m}$, equal the corresponding ratios in the prototype, $\Lambda_{i/p}$ (i.e. $\Lambda_{i,m}=\Lambda_{i/p}$ for all $i=1,\ldots,j$) the model and prototype are said to be geometrically similar. If all corresponding arguments of F are equal for model and prototype, the model and prototype are said to be dynamically similar. If any of the restrictions specified at the beginning of this section are relaxed, additional variables accounting for the physical phenomena introduced must be included in the analysis.

Often one of the standard equivalent diameters [3] is chosen to transform a distribution of non-spherical particles (which in general

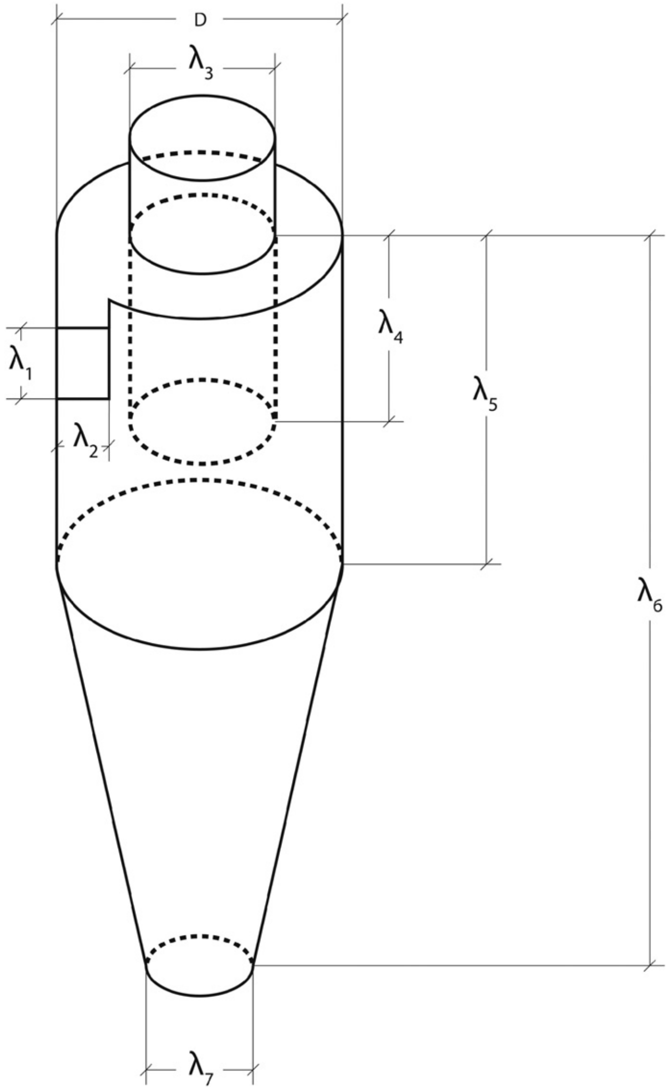

Fig. 1. A reverse flow cyclone separator shown with typical design lengths λ_i.

require more than one length to specify) into a distribution specified by an equivalent diameter d, or such a transformation is imposed by the particle sizing apparatus. Evidently η and Ω depend on whether such a transformation is applied and, if so, which equivalent diameter is chosen or imposed. The following discussion is limited to joint probability distributions of the form $\Omega = (d, \rho_p; \kappa_1,\ldots,_k)$, where ρ_p is the particle density and the κ_i are distribution parameters, or $\Omega = \Omega(\delta, \rho_p/\rho_f; K_1,\ldots,K_l)$, where δ is a dimensionless equivalent diameter, ρ_p/ρ_f is the particle-fluid density ratio, the K_i are the corresponding dimensionless distribution parameters and $l \le k$. If all particles to be separated have the same density, we write $\Omega_d = \Omega_d(d; \kappa_1,\ldots,\kappa_k)$ or $\Omega_\delta = \Omega_\delta(\delta; K_1,\ldots,K_l)$ if ρ_p/ρ_f is the same for all particles. It is useful to note

$$\Omega_\delta = \int_0^\delta \omega_x dx$$
$$\frac{d}{d\delta}\int_0^\delta \omega_x dx = \omega_\delta.$$

3. Discussion

Gallaer and Schindeler [4] treated the case where $\eta = 1 - \exp.(-\alpha d)$, $\Omega_d = 1 - \exp.(-\beta d)$, $0 < d < \infty$, so $\Omega = \Omega(d, \rho_p; \beta)$, with one fixed ρ_p. The overall height of the cyclones they considered was between 5.5 and 9.1 m, so $D \sim 2$ m. Since β does not depend on cyclone design or operating conditions while α does, let $\delta = \beta d$. The functional relationship among $\overline{\eta}$, η, and Ω_δ is known:

$$\overline{\eta} = \int_0^\infty \eta\ \omega_x dx = \int_0^\infty \left(1-e^{-\frac{\alpha}{\beta}x}\right)e^{-x}dx = \frac{\alpha/\beta}{1+\alpha/\beta}$$

F becomes

$$F_{(G-S)}\left(N_{\mathrm{Re}}, \varepsilon, \Lambda_i, \ldots, \Lambda_j, \alpha/\beta, (\beta D)^{-1}, \rho_p/\rho_f, \overline{\eta}, N_{\Delta\mathrm{p}}\right) = 0.$$

The loci of constant α/β are loci of constant $\overline{\eta}$. The authors reported that for the conditions they studied, $N_{\Delta p}$ depended only on the Λ_i; thus for a given design $N_{\Delta p}$ was a constant.

Theodore and DePaola [5] proposed a two parameter model for fractional efficiency:

$$\eta = \frac{(d/d_{50})^\psi}{1 + (d/d_{50})^\psi}$$

$d > 0$, $\psi > 0$; at $d = d_{50}$, $\eta = 0.5$. As $d \longrightarrow \infty$, $\eta \longrightarrow 1$ asymptotically. If $\psi \le 1$, η is concave down everywhere. If $\psi > 1$, there is an inflection point at $d/d_{50} = [(\psi + 1)/(\psi - 1)]^{-1/\psi}$ at or below which η is concave up and above which η is concave down. As $\psi \longrightarrow \infty$, $[(\psi + 1)/(\psi - 1)]^{-1/\psi} \longrightarrow 1$ and the cut becomes sharper. The authors found the model satisfactorily fit smoothed laboratory and in-plant data provided by Lapple [6] when $\psi = 2$.

Iozia and Leith [7] used this model to fit fractional efficiency data obtained from cyclones of various designs ($D = 250 \times 10^{-3}$ m) using a mineral oil aerosol ($\rho_p = 876$ kg·m^{-3}) at very low mass loading ($1 - \varepsilon \sim 10^{-5}$ [8]) suspended in air. Tests at three volumetric flow rates were made with a Stairmand high efficiency design [9]. Eight additional tests were made at a fixed volumetric flow rate with certain systematic variations of the λ_i from the Stairmand design. Ω_d and $\overline{\eta}$ were not reported; because of this the similitude problem was inadequately specified. Let d_c denote a characteristic equivalent diameter of Ω and define $\delta = d/d_c$. Then

$$\overline{\eta} = \int_0^\infty \frac{[(d_c/d_{50})x]^\psi}{1 + [(d_c/d_{50})x]^\psi}\omega_x dx$$

and

$$F_{(I-L)}\left(N_{Re}, \varepsilon, \Lambda_1, \ldots, \Lambda_6, d_c/d_{50}, \psi, d_c/D, \rho_p/\rho_f, \Omega_\delta, \overline{\eta}, N_{\Delta\mathrm{p}}\right) = 0.$$

The experiment was executed at ambient conditions which were not specified. For discussion purposes, suppose the discharge pressure and temperature were 100 kPa and 288 K respectively. Using these values, N_{Re} and $N_{\Delta p}$ were calculated from reported data and were used to construct Table 1. Averaged over all test numbers, $\rho_p/\rho_f = 722 \pm 5$; the

Table 1
Cyclone performance data [7].

Test	$N_{Re} \times 10^{-3}$	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	ψ	d_{50} [μm]	$N_{\Delta p}$
1	258	0.5	0.2	0.5	0.5	1.5	4	2.77	3.65	2.86
2	127	0.5	0.2	0.5	0.5	1.5	4	2.15	4.91	2.93
3	388	0.5	0.2	0.5	0.5	1.5	4	3.82	2.71	3.18
4	172	0.5	0.3	0.5	0.5	1.5	4	3.14	4.13	5.23
5	172	0.75	0.2	0.5	0.75	1.5	4	4.11	4.46	5.23
6	519	0.5	0.1	0.5	0.5	1.5	4	5.22	2.40	1.95
7	518	0.25	0.2	0.5	0.25	1.5	4	5.03	2.90	1.51
8	260	0.5	0.2	0.3	0.5	1.5	4	4.82	2.23	7.80
9	257	0.5	0.2	0.7	0.5	1.5	4	2.83	4.34	1.61
10	258	0.5	0.2	0.5	0.5	0.5	3	3.42	3.34	3.39
11	258	0.5	0.2	0.5	0.5	2.5	5	3.27	3.23	2.86