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Study on Fractal Mathematical Models of Pulverizing Theory for Ore

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ABSTRACT

Fractal dimension is an important parameter to characterize size distribution and shape features of particles. However, it still remains unclear whether the fractal dimensions measured with different methods for the same particles will have the identical results, or whether the similar quantitative relationship exists between D_3 and D_2 or D_3 and D_1 as stated in the product–sum theorem. Silicon dioxide and potassium feldspar were selected for PDS measurements and 2DSPI extractions in this paper. By introducing the concept of fractal size, D_3, D_2 and D_1 are defined in a unified way, and their relationships are obtained under the defined assumptions including the product–sum theorem. The two fractional mathematical models and the three new algorithms including the power spectrum method presented in this paper provide the solutions to the problems encountered in the measurement of fractal dimensions. The Gauss–Newton iterative method of R–R distribution is superior to the existing fitted method. The four relationships of the fractal dimensions have been verified both experimentally and by the computational models. The results show that the relationships among D_3, D_2 and D_1 are rational and valid.

1. Introduction

A self-similar system is called a fractal [1]. The fractal dimension has become an important tool for characterizing objects, forms and surfaces in some areas of science in the past three decades [2]. Its applications can be found as widely as in nanoparticles [3–5], surface roughness [6], aggregates [7–9], porous materials [10], floc [11], waste printed circuit boards [12] and particle surfaces [13]. A non-integer dimension is usually defined as the fractal dimension in the engineering practice. Earlier publications did not clearly differentiate the specific fractal dimensions. In his paper, Bruce E. Logan categorized the fractal dimensions of particles as D_1, D_2 and D_3 which are related to the perimeter, cross-sectional area and solid volume of particles respectively [14]. A. Helalizadeh identified the range for the surface fractal dimension as $2 < D_2 < 3$, and the volume fractal dimension as $1 < D_3 < 3$ [15]. In this paper, D_3 represents the volume fractal dimension describing the characteristic of particle size distributions (PSDs). D_2 from Power spectrum method is the surface fractal dimension for the surface of a single particle by its profile, and D_1 from Yardstick method represents the line fractal dimension for the two-dimensional single particle image (2DSPI) profile.

Most particles satisfy the statistical self-similarity and belong to irregular fractals. The fractal dimension is considered as a useful parameter to characterize the complexity of 2DSPIs. The measurement methods for fractal dimensions are usually diverse in the practical applications. A. G. Flook measured the fractal dimensions of carbon agglomerates by the yardstick method, in which the relationship between the exponent and the fractal dimension is $\alpha = 1 - D_1$ [16,17]. Later, the power spectrum method (also known as the spectral method), which is recognized as one of the best models with the least independent parameters to describe natural surface geometry qualitatively and quantitatively [18–20], was widely applied in fractal studies of machined surfaces and spectroscopic. Generally, the yardstick method requires a lot of 2DSPIs to calculate the line fractal dimension in practical applications. The fractal dimension model for PSDs comes from the Rosin–Rammler (R–R) distribution or the Gaudin–Schuhmann (G–S) distribution. Li et.al studied the fractal dimensions of the particle images and G–S distributions from the rock comminution [21,22], and noted that $D_3 \approx D_2$ for brittle materials with no abnormal cleavages [23]. Xie proposed a fractal model of R–R distribution [24]. The exponent α in G–S distribution and the uniformity coefficient n in R–R distribution are both equal to $3 - D_3$. Yang studied the relationship between fractal dimensions and the ultrafine coal powder from ball milling processes and obtained that the maximum fractal dimension of PSDs as 1.02 and the minimum as −0.40 for different dispersants [25,26].

Upon reviewing the previous work on fractal dimensions, it still remains unclear whether the fractal dimensions measured with different methods for the same particles will have the identical results, or whether the similar quantitative relationship exists between D_3 and D_2 or D_3 and D_1 as stated in the product–sum theorem [27] (also known as Mandelbrot experience theorem, i.e. $D_2 = D_1 + 1$).

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It is also unclear whether the relationships among D_3, D_2 and D_1 can be experimentally verified.

2. The definition and mathematical models of fractal dimensions for particles

2.1. The definition of fractal dimensions for particles

If the density for dimensionless coordinates $\vec{x}$ for the randomly distributed particles in a given space is expressed as $\rho(\vec{x})$, then the correlation function $c(\vec{r})$ can be defined as:

$$c\left(\vec{r}\right) \equiv \left\langle \rho\left(\vec{x}\right)\rho\left(\vec{x}+\vec{r}\right)\right\rangle. \tag{1}$$

Where $\langle \ldots\ldots \rangle$ denotes average. If the distribution in all directions is equal, the correlation function can be expressed by the function of the distance between the two points r. When the random distribution satisfies the fractal conditions, the correlation function can only take the power-law forms [28], i.e.

$$r = \left|\vec{r}\right| \quad , \quad c(r) \propto \begin{matrix} r^{\alpha} & \alpha<0 \\ r^{-\alpha} & \alpha>0 \end{matrix} \Bigg\}. \tag{2}$$

The fractal dimension D_i is defined as:

$$D_i = d_i - \alpha \qquad \begin{matrix} \alpha<0, & i=1,\ 2 \\ \alpha>0, & i=3 \end{matrix} \Bigg\}. \tag{3}$$

Where d_i is the Euclidean space dimensions 1, 2 and 3 respectively; Correspondently, D_i represents the line fractal dimension D_1 $(1<D_1<2)$ [1], the surface fractal dimension D_2 $(2<D_2<3)$ and the volume fractal dimension D_3 $(1<D_3<3)$. α is the power exponent.

Assume that there exists the correlation function $c(r_F)$, and the dimensionless coordinates $\vec{x}_F$ in fractal sets, and they are the one-to-one correspondence to the correlation function $c(r)$ and dimensionless coordinates $\vec{x}$ in Euclidean space respectively. The correlation function $c(r_F)$ can only take the power–law forms:

$$r_F = \left|\vec{r}_F\right| \quad , \quad c(r_F) \propto \begin{matrix} r_F^{\alpha} & \alpha<0 \\ r_F^{-\alpha} & \alpha>0 \end{matrix} \Bigg\}. \tag{4}$$

Where, r_F is the distance between the two points in a fractal set. When $i=1,2$, the correlation functions $c(r)$ and $c(r_F)$, both of which satisfy the irregular fractal conditions, are self-similar with the similar ratio r^{-1}, then

$$r^{2-D_1} \propto r_F^{1-D_1}. \tag{5}$$

$$r^{3-D_2} \propto r_F^{2-D_2}. \tag{6}$$

What Eqs. (5) and (6) have in common is that the sum of the power exponents equals to the regional value of 1 in their fractal dimension domain, which is the same as the characteristic number of the point, the line and the surface [28]. When $i=3$, the correlation functions $c(r)$ and $c(r_F)$, both of which satisfy the irregular fractal conditions, are self-similar, however, their similar ratio is r^{-2}, then

$$r^{D_3-1} \propto r_F^{D_3-3}. \tag{7}$$

In Eq. (7), the sum of the power exponents still equals the regional value of 2 in its fractal dimension domain, and it is also equal to the Euclidean number of the polyhedron [29]. Eqs. (8), (9) and (10) have been formulated using Eq. (5), (6) and (7).

$$r_F^{1-D_1} = k_1 r^{2-D_1} \qquad \log r_F = \frac{2-D_1}{1-D_1}\log r + \frac{1}{1-D_1}\log k_1 \tag{8}$$

$$r_F^{2-D_2} = k_2 r^{3-D_2} \ \text{i.e} \quad \log r_F = \frac{3-D_2}{2-D_2}\log r + \frac{1}{2-D_2}\log k_2 \tag{9}$$

$$r_F^{D_3-3} = k_3 r^{D_3-1} \qquad \log r_F = \frac{D_3-1}{D_3-3}\log r + \frac{1}{D_3-3}\log k_3 \tag{10}$$

Since all three fractal dimensions are unique for a given particle, the necessary and sufficient conditions that satisfy the product–sum theorem from Eqs. (8) and (9) are

$$\frac{2-D_1}{1-D_1} = \frac{3-D_2}{2-D_2}, \qquad \frac{1}{1-D_1} = \frac{1}{2-D_2}. \tag{11}$$

Then $k_1 = k_2$. Because the correspondence between r and r_F is one to one, the two straight lines are coincident in the double logarithmic coordinates. Suppose, in the double logarithmic coordinates, the three lines including the volume fractal dimension D_3 are all coincident, and let

$$k = {k_1}^{1/(1-D_1)} = {k_2}^{1/(2-D_2)} = {k_3}^{1/(D_3-3)}. \tag{12}$$

Then, the relationships between r_F and r can be represented as:

$$r_F = k r^{(2-D_1)/(1-D_1)} = k r^{(3-D_2)/(2-D_2)} = k r^{(D_3-1)/(D_3-3)}. \tag{13}$$

Since the exponents in the equations above are all negative, the reciprocal of r_F is defined as fractal size s_F, therefore $s_F = r_F^{-1}$, which reflects the hierarchy of the self-similarity. The mathematical models for D_3 and D_1 were shown in Section 2.2. Eq. (14) has been formulated using Eq. (13).

$$\frac{2-D_1}{1-D_1} = \frac{3-D_2}{2-D_2} = \frac{D_3-1}{D_3-3} \tag{14}$$

Thus the relationships among the three fractal dimensions should be:

1) The relationship between D_3 and D_2 is:

$$D_3 = 7-2D_2. \tag{15}$$

2) The relationship between D_3 and D_1 is:

$$D_3 = 5-2D_1. \tag{16}$$

3) The relationship between D_2 and D_1 is:

$$D_2 = D_1 + 1. \tag{17}$$

4) The relationship among D_3, D_2 and D_1 is:

$$D_3 + D_2 + D_1 = 6. \tag{18}$$