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Multiwavelets and multiwavelet packets of Legendre functions in the direct method for solving variational problems

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ABSTRACT

A numerical technique for solving the linear problems of the calculus of variations is presented in this paper. Multiwavelets and multiwavelet packets of Legendre functions are used as basis functions in the Ritz method of formulation. An operational matrix of integration of multiwavelets and multiwavelet packets is introduced and is used to reduce the calculus of variation problem to the solution of the system of algebraic equations. The algorithm is applied to the analysis of mechanic problems which are formulated as functionals. Two examples are considered in this paper. The first example concerns the stability problem of a Euler–Bernoulli beam and the second one presents the calculation of the extreme value of the functional which defines the potential energy of an elastic string. The presented method yields the approximate solutions which are convergent to accurate results.

1. Introduction

Variational principles are widely used in various fields of science: physics, mathematics and electrical engineering as well as many others [10,13,20]. The basic principles of variational calculus in the mechanics of systems and structures in which the volume and surface forces are in a state of equilibrium include, among others Lagrange's variational principle and the Castigliano's principle [13,20]. These principles are based on the theorem of minimum potential energy. A stationary condition of the functional which describes the potential energy of the system expressed in the form of displacements (Lagrange's principle) or stresses (Castigliano's principle) which are represented by independent variables further leads to an admissible state of displacement and stress of the analyzed system [13,20].

Direct methods for solving the variational problems such as Ritz's, Kantorowicz's, Bubonow-Galerkin's, Treffz's and any other, generally reduce the issue to a system of algebraic equations for which the solution is given by the coefficients of assumed approximation [10,13]. By assuming the approximation functions which are kinematically admissible, the condition for the functional stationarity written in the form

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of potential energy or virtual work of the system, enables the approximate solution of a given problem to be obtained [12].

The advantage of variational formulation is that the integral variational principles define the problem globally. Generally, in the functional form all equations are set describing the phenomenon together with the boundary conditions that did not exist at the local (differential) formulations.

Operational matrices of integration were first used in the direct methods for solving variational problems by Chen and Hsiao [4] by the taking of Walsh functions as the basis functions. Initial works that have given rise to the development of methods in which the integral operators are represented in the basis of orthogonal functions and have shown the application of the above method in different problems of science [8,5]. A solution to the heat flow problem in the variational formulation using operational matrices of Haar functions has been presented by Hsiao [15]. Glabisz [11,12] has developed a procedure that allows the solution of the variational problem to be obtained with operational matrix of integration of Walsh-wavelet packet functions and by introducing a dual approximation field, has also shown the solution of an extreme functional by assuming an additional internal condition. Razzaghi and Yousefi [23] presented the solution of the convection problem and as an example of optimal control solved the issue of brachistochrone [22]. In both cases, the authors of the work [24,22] used the operational matrices of integration of Legendre multiwavelet functions, for which the algorithm of creation has been presented [23]. Legendre multiwavelet functions have been used to find the extreme values of the functional by Khellat and Yousefi [19]. Sadek et al. [26] successfully used Legendre multiwavelets in the problem of optimal control of transverse vibration of a Euler Bernoulli beams.

So far, multiwavelet bases have been rarely used in the analysis of mathematical models describing the problems of structural mechanics, and multiwavelet packets in such issues have not been used at all. In the article an algorithm for the solution of exemplary variational problems based on Ritz's direct method using the operational matrix of integration of Legendre multiwavelets and multiwavelet packets is presented. Two examples are analyzed. The first example concerns the stability of the equilibrium elastic simple beam and the second concerns the calculation of the deflection line of string which has a finite length. The influence of the form of the operational matrix of integration on the accuracy of the results is analyzed. The purpose of this article is to show that Legendre multiwavelets and Legendre multiwavelet packets taken as approximation functions can be an alternative to other basis functions such as Chebyshew functions [14], Laguerre [17], Jacobi series [21], and Haar functions [15,16] which have usually been used to obtain the solutions of such variational issues.

In Section 2 basic information on classical and packet multiwavelet analysis is provided. The definition of the integral operational matrices of multiwavelet functions and multiwavelet packets based on Legendre functions is presented in Section 3. The formulation of the problem under consideration is given in Section 4. The application of an algorithm for determining the solutions of the discussed issues is presented in Section 5. Section 6 is a summary of the obtained results.

2. Multiwavelets and multiwavelet packets of Legendre functions

In the multiwavelet analysis the set $k2^j$ of scaling functions $\varphi_{jl}^n(x)$ define the space V_j^k and the set of $k2^j$ wavelet functions $\psi_{jl}^n(x)$ define space W_j^k [1–3,6,18,27]. Basis functions $\varphi_{jl}^n(x)$ and $\psi_{jl}^n(x)$ are obtained by dilation ($2^{j/2}$) and translation (l) of each from k functions which generate the set of fundamental multiscaling functions $\varphi^n(x) = \{\varphi^0(x), \varphi^1(x), \ldots, \varphi^{k-1}(x)\}$ from the space V_0^k and fundamental multiwavelets functions $\psi^n(x) = \{\psi^0(x), \psi^1(x), \ldots, \psi^{k-1}(x)\}$ from the space W_0^k satisfying the following relations

$$\varphi_{jl}^n(x) = 2^{j/2}\varphi^n(2^j x - l), \quad x \in [2^{-j}l, 2^{-j}(l+1)n_s] \tag{1}$$

$$\psi_{jl}^n(x) = 2^{j/2}\psi^n(2^j x - l), \quad x \in [2^{-j}l, 2^{-j}(l+1)n_f] \tag{2}$$

where n_s is the compact support length of fundamental multiscaling functions, n_f is the compact support length of fundamental multiwavelet functions, $n = 0, \ldots, k-1, l = 0, \ldots, (2^j - 1)n_{s,f}$, j denotes the level of approximation and parameter k determines the number of fundamental multiwavelet function sets as well as the number of fundamental multiscaling function sets. Number k does not depend on the assumed level j.

The set of multiwavelet functions $\psi^n(x) = \{\psi^0(x), \psi^1(x), \ldots, \psi^{k-1}(x)\}$ is generated on the basis of multiwavelet scaling functions $\varphi^n(x) = \{\varphi^0(x), \varphi^1(x), \ldots, \varphi^{k-1}(x)\}$ [1,2,9,27]. The equations which define the functions $\psi^n(x)$ and $\varphi^n(x)$ take the following forms

$$\psi^n(x) = \sqrt{2}\sum_{j=0}^{k-1}(g_{n,j}^{(0)}\varphi^j(2x) + g_{n,j}^{(1)}\varphi^j(2x-1)), \quad n = 0, \ldots, k-1 \tag{3}$$

$$\varphi^n(x) = \sqrt{2}\sum_{j=0}^{k-1}(h_{n,j}^{(0)}\varphi^j(2x) + h_{n,j}^{(1)}\varphi^j(2x-1)), \quad n = 0, \ldots, k-1 \tag{4}$$

Multiwavelet functions $\psi^n(x)$ and scaling functions $\varphi^n(x)$ defined by the relations (3) and (4) are presented as linear combinations of the same functions, but their differences determine respectively low and high-pass filter coefficients $\{h_{n,j}^{(0)}, h_{n,j}^{(1)}, g_{n,j}^{(0)}, g_{n,j}^{(1)}\}$.

The function space W_j^k created by multiwavelets $\psi_{jl}^n(x)$ is orthogonal complementation of the function space V_j^k to the space of the upper level of approximation V_{j+1}^k and can be defined as

$$\begin{aligned} V_{j+1}^k &= V_j^k \oplus W_j^k = V_{j-1}^k \oplus W_{j-1}^k \oplus W_j^k \\ &= V_{j-m}^k \oplus W_{j-m}^k \oplus \ldots \oplus W_{j-1}^k \oplus W_j^k \end{aligned} \tag{5}$$

According to expression (5), any square integrable function from the j level of approximation, when the multiscaling functions are taken from the m level of approximation, can be expressed in multiwavelet bases [9] in the following formula

$$f_m^j(x) = \sum_{l=0}^{2^m-1}\sum_{n=0}^{k-1}s_{m,l}^n\varphi_{m,l}^n(x) + \sum_{l=0}^{2^v-1}\sum_{v=0}^{j-1}\sum_{n=0}^{k-1}d_{v,l}^n\psi_{v,l}^n(x) \tag{6}$$