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Sensitivity analysis in PERT networks: Does activity duration distribution matter?

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ABSTRACT

The Program Evaluation and Review Technique (PERT) is a stochastic network technique developed in the late 1950s. The original model assumes PERT β distribution for the activity durations. Later on, this was criticized by many researchers, and several new distribution types have been introduced, which were believed to be better in modelling the real activity duration distributions or were easier to handle from a mathematical point of view. Introduction of new activity duration distributions was criticized even by Clark—one of the pioneers of PERT—while other researchers were arguing that these new activity distributions could better describe the stochastic distributions of the activity durations and their application helps define the distribution of project duration better. In the course of our research, we have investigated the effect of various activity duration distribution types (PERT-beta, uniform, triangular, lognormal) on the project duration, as well as the impact of the inaccuracy of the activity durations' estimation when performing the PERT three-point estimation. Our basic assumption—that the differences in the distribution of the project duration caused by using different activity duration distributions are not considerable compared to the differences caused by the inaccuracy of the three-point estimations—has been tested on several hypothetical projects and case studies. Four different distributions have been applied, one at a time. The aim has been to clearly show the differences between the results of the application of the selected distributions, therefore the distributions have not been mixed. Monte Carlo analysis has been used to create the probabilistic distributions of the projects.

All example projects have proved the basic assumption to be true. The results suggest that from a practical point of view, the accuracy of the three-point estimation is more important than the type of the activity duration distribution. Consequently, the selected activity duration distribution is essentially insignificant. The most important practical consequence of this research is that instead of the time-consuming and costly process of selecting the proper activity duration distributions, planners should devote more effort to adequately determine the activity durations.

1. Introduction and the rationale of the study

An important step in creating any PERT network is the definition of the probabilistic distributions of the activity durations. This step requires the definition of the distribution type of the activity duration and the definition of the adherent durations usually using three-point estimation. Both the definition of the activity durations and the definition of the distribution types can be sources of numerous errors that could lead to an unrealistic final result. To the best of our knowledge, there has been no research on the effects of these potentially erroneously accomplished steps on the distribution of the project duration, and the relative weights of these errors have never been compared. In this paper, the effect of the application of different activity duration distributions on the distribution of the project duration is examined in PERT networks. The basic assumption is that the difference between the results caused by different activity duration distributions is not significant compared to what a 10% inaccuracy in the estimation of the most likely, optimistic and pessimistic values of the activities can cause to the distribution of the project duration. A precision level of 10% (i.e. estimating the most likely activity duration to be 90 or 110 weeks instead of 100 for the most likely duration, and the same precision level for the optimistic and pessimistic durations) can be considered acceptable, since in reality the estimation of the activity durations is usually less accurate, or at least there is no proof for the opposite. Therefore, if such a slight difference can affect the results to a greater extent than the type of the activity duration distribution can, then, from a practical point of view, it is unimportant which distribution type is chosen.

The second section provides a short introduction to the Program Evaluation and Review Technique (PERT). In the third section, the most important activity duration distributions and the criticism they have received are discussed. Section four shows the results obtained

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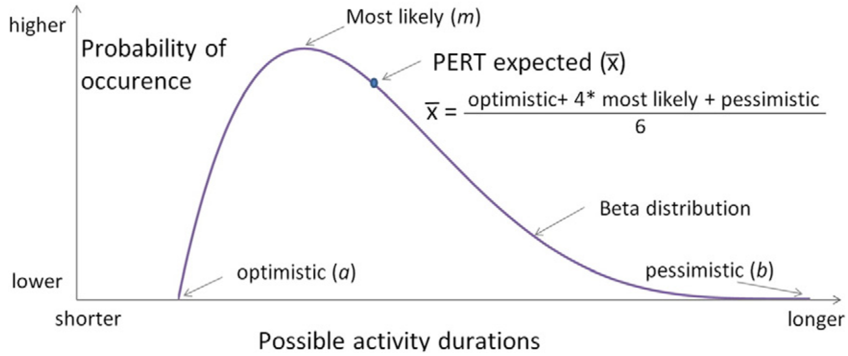

Fig. 1. Typical density function of the PERT-beta distribution.

by testing the basic assumption on example projects. First, three hypothetical networks are examined, starting with the simplest and continuing with the more complex ones. Secondly, four real-life (three infrastructure and one bridge construction) projects are analysed. The last section provides the conclusions of the research.

2. PERT basics

The original Program Evaluation and Review Technique (PERT) [1] is an activity-on-arrow network with one start and one finish event, which represents the beginning and the end of a project. To accomplish the project, certain activities must be carried out according to a given pre-defined sequence. This logic is depicted by a directed, acyclic graph in which the vertices of the graph represent the events, while the arrows represent the tasks to be performed. An event occurs when all preceding activities have been completed; only then can the succeeding tasks start. The event is basically used for expressing logical dependencies between activities.

In a PERT network, activity durations are defined by stochastic variables that are assumed to be independent of each other. The distribution of the activity durations follows a so-called PERT-beta distribution. The formula of the beta function is shown below (Eq. (1). In the formula, α and β are the parameters of the beta distribution; while a and b are the endpoints of the domain of x. Outside the interval, $f(x) = 0$. The distribution is identified as PERT-beta if the α and β parameters of Eq. (1) are greater than 1 ($\alpha > 1$ and $\beta > 1$). This ensures that $f(x)$ has one maximum, and $f(x)$ tends to zero at the endpoints of the domain $f(a) = f(b) = 0$ (Fig. 2).

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha-\beta-1}}, \qquad a<x<b, \qquad \alpha,\beta>0 \tag{1}$$

The mean ($\bar{x}$) and variance (σ_x^2) of the activity durations in PERT are defined according to Eqs. (2) and (3), respectively:

$$\bar{x} = \frac{a+4m+b}{6} \tag{2}$$

$$\sigma_x^2 = \left(\frac{b-a}{6}\right)^2 \tag{3}$$

where a, m and b are subjective values, determined by a specialist, representing the optimistic (a), most likely (m) and pessimistic (b) durations of the activity (see Fig. 1). The process of defining these subjective values is called the PERT three-point estimation method.

The main goal of the PERT analysis is to create a distribution of the project duration.

According to PERT theory, the project duration follows a normal distribution, with the mean being the result of time analysis based on activity mean durations ($\bar{x}$) and the variance being equal to

$$\sigma_{PD}^2 = \sum_{x\in CP} \sigma_x^2 \tag{4}$$

where σ_{PD}^2 is the variance of the distribution of the project duration (PD) and x represents the activities on the critical path (CP). These calculations are based on the central limit theorem of mathematical statistics.

The theoretical optimistic and pessimistic project durations, that is, the lower and upper bounds of the distribution of the project duration can be defined as the results of time analysis performed with the optimistic and pessimistic values, respectively.

3. Reviews and developments of PERT

The use of the beta distribution in PERT has been criticized by many researchers. Researchers have introduced different distributions, like the uniform [2,3], the mixed beta and uniform [4], the triangular [5,6], the trapezoidal [7], the general trapezoidal [8], the gamma [9,10], the lognormal [11], the exponential [12], the beta rectangular [4], the doubly truncated normal [13], the Parkinson [14], the generalized biparabolic [15] and the tilted beta distribution [16], among others. Some authors—among them Clark [17]—argue against the introduction of new probability distributions into PERT. According to him "*The author has no information concerning distributions of activity times*; *in particular, it is not suggested that the beta or any other distribution is appropriate*." In the same line of thinking, Kamburowski [18] stands by the applicability of the original assumptions (Eqs. (2) and (3)) and opposes those who believe that a different distribution must be introduced. He argues that due to the significant uncertainty and imprecision reflected in the estimates, the precision that we can achieve using beta distribution for activity durations is satisfactory. However, these statements—according to the best knowledge of the authors—have never been justified in the necessary depth. One of the main purposes of this paper is to justify the above quotation by showing that the usage of different distributions does not result in considerably great differences in the distribution of the project duration, or at least they are smaller than the difference caused by a 10% inaccuracy in the estimation of the most likely values of the activities. Such kinds of investigations were initiated earlier by the authors [19]; however, the set of activity distributions used in their previous study (beta, uniform, triangular) is widened here with

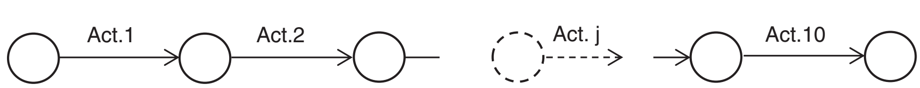

Fig. 2. Sample project #1.