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Koiter asymptotic analysis of multilayered composite structures using mixed solid-shell finite elements

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A B S T R A C T

In this paper we propose a powerful tool for the evaluation of the initial post-buckling behavior of multilayered composite shells and beams in both bifurcation and limit load cases, including mode interaction and imperfection sensitivity. This tool, based on the joint use of a specialized Koiter asymptotic method and a mixed solid-shell finite element model, is accurate, simple and characterized by a computational cost far lower than standard path-following approaches and many advantages with respect to asymptotic analysis performed with shell elements. The method is very simple and easy to include in existing FE codes because it is based on the same ingredients of a linearized buckling analysis, with very light formula due to the presence of displacement degrees of freedom only. Due to its efficiency it is suitable for layup design when geometrical nonlinearities have to be considered.

1. Introduction

Multi-layered composite structures are an important and growing feature of engineering construction with the areas of application becoming increasingly diverse, ranging from aerospace and mechanical to civil engineering. Many factors have contributed to this growth, among which is the weight economy that requires the use of optimized thin-walled structures for which buckling often becomes the design constraint. As a consequence of the design process the structural behavior is often dominated by coincident, or almost coincident, buckling loads and by buckling mode interaction phenomena with a potentially, strongly unstable post-critical behavior. In light of this a *sensitivity analysis* [1–3], that is the evaluation of the limit load for a set of possible *external imperfections*, becomes mandatory. Furthermore the layup optimization requires a large number of numerical experiments to consider not only the linear and the buckling behavior but, more correctly, the post-critical one, including the limit load evaluation for the worst imperfection case [4–6]. Standard path-following approaches, aimed at recovering the equilibrium path for a single loading case and assigned imperfections, are not suitable for this purpose due to the high computational burden of the single run and being unusable if no "a priori" information about the worst imperfection shapes is available.

The asymptotic approach, derived as a finite element (FE) implementation [7–9] of the Koiter nonlinear theory of elastic stability [10], can be a convenient alternative as it provides an effective and reliable strategy for predicting the initial post-critical behavior in both cases of limit or bifurcation points, including *modal interactions* [11–13]. It has also been applied to the study of composites in [14,15]. An extension to dynamic effects can be found in [16]. The most interesting feature of the method is that, once the analysis of the so-called *perfect structure* has been performed, the presence of small loading imperfections or geometrical defects can be taken into account in a post-processing phase with a negligible computational extra-cost, so allowing an inexpensive imperfection sensitivity analysis [6,4,5]. It is also possible to obtain information about the worst imperfection shapes [17], and it can be used to improve the imperfection sensitivity analysis or to address more detailed investigations through specialized path-following analysis [18–20]. The accuracy of the method has been confirmed by numerical testings and theoretical investigations [21] but it requires great care in both the mechanical modeling [22,23] and finite element implementation to avoid: (i) *interpolation locking* phenomena in the evaluation of the energy variation terms [24,25,1]; (ii) *extrapolation locking* phenomena [26,27] due to an inappropriate format used in the control variables.

Although this great amount of work aimed to make the method a general tool in a finite element context suitable for practical design, a series of limitations still exists. In particular for geometrically exact [22,23] or corotational [28,29] beam and shell models that use 3D finite rotations [30], the asymptotic

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analysis is highly penalized by the complex and expensive expressions of the high order strain energy variations. In this context they are so complex to evaluate [31] that usually simplified assumptions on the kinematics or on the precritical behavior are necessary leading, however, to a not "a priori" evaluable loss of accuracy. Furthermore, as the method is based on a fourth order energy expansion, in the multimodal case it requires the evaluation of a number of strain energy variations proportional to m^4 where m is the number of buckling modes considered. Consequently the computational cost of the Koiter analysis quickly grows with m.

Although the formulation is a very suitable tool for sensitivity analysis these limits influence its performances in terms of both computational cost and accuracy and have penalized its diffusion in the scientific community and in the commercial codes.

In recent years an increasing amount of research has aimed at developing new efficient solid-shell finite elements [32,33] for the linear and nonlinear analysis of thin structures. This is due to the advantages of this kind of elements in comparison to classical shell ones. In particular they allow the use of the 3D continuum strain measures employing translational degrees of freedom only [34–36] and so the avoidance of complex and expensive rules for updating the rotations. In particular, the Green–Lagrange strain measure, describes the structural behavior coherently through a low order dependence on the displacement field, and consequently gives simpler expressions for the strain energy and its variations in comparison with beam and shell models [22,23,37]. To maintain an acceptable number of degrees of freedom, the solid-shell elements are usually based on a low order displacement interpolation. Consequently they show locking phenomena: the shear and membrane locking also present in classical shell elements and trapezoidal and thickness locking, typical of low order solid-shell elements [38]. These lockings are usually sanitized by means of Assumed Natural Strain, Enhanced Assumed Strain [39–41] and mixed (hybrid) formulations [32,42,43]. In this way solid-shell elements have now reached a high level of efficiency and accuracy and have also been used to model composites or laminated beams [40,42,44] and shell structures in both the linear [39,35,45] and nonlinear [41,34,32] range. Among the most effective and interesting proposals there are the mixed solid-shell elements of Sze and co-authors [46–48] which extend the initial PT18β hybrid element of Pian and Tong to thin shell.

The aim of this paper is to overcome the limitations previously described making the Koiter analysis a powerful tool for the evaluation of the initial post-buckling behavior of multi-layered composite shells and beams in both cases of bifurcation and limit load, including mode interaction and imperfection sensitivity. This tool, based on the joint use of a specialized Koiter asymptotic method and a mixed solid-shell finite element model with a thickness pre-integration, is accurate, simple and characterized by a computational cost far lower than standard path-following approaches and has many advantages with respect to asymptotic analysis performed with shell elements.

The use of a mixed solid-shell element based on the quadratic Green–Lagrange strain measure allows the representation of the strain energy as a third order polynomial function of the finite element variables. Exploiting this property, it is possible to develop a new asymptotic algorithm which is more accurate and computationally efficient, than that based on classical shell elements because: (i) an exactly linear buckling condition is obtained without the need of any simplified assumption on the kinematics or the precritical behavior, a part from the linear extrapolation of the fundamental path; (ii) the simple an low cost expressions of the strain energy variations with the fourth order ones which are exactly zero; (iii) a more coherent reduced nonlinear equations used for the evaluation of the equilibrium path.

It is important to note as the linear extrapolation of the fundamental path gives completely different results when a mixed (stress and displacement) or a displacement description of the problem are adopted. The mixed description allows, in fact, the elimination of the extrapolation locking phenomenon discussed in detail in [49,26] in the context of path-following analysis and in [31,13,50] for asymptotic analyses. The extrapolation locking affects any displacement model and consists in an overestimated stiffness evaluated in an extrapolated point. The mixed formulation, not affected by this phenomenon, ensures: in path-following analysis, a faster convergence of the Newton (Riks) iterative process, noted by many authors (see for example [32,34]); in asymptotic analysis, a more accurate linearized buckling analysis and then an accurate recovery of the equilibrium path, even when the precritical behavior is not linear.

Finally it is worth mentioning that the use of both displacement and stress variables increases the dimension of the problem, but generally the computational extra-cost, with respect to a displacement formulation, is very low. In fact performing a static condensation of the stress variables, locally defined at element level, the global operations involve displacement dofs only. The resulting small computational extra-cost is largely compensated by the zeroing of the computationally expensive fourth order strain energy variations.

The proposed method is very simple and easy to use in existing FE codes because it is based on the same ingredient, that is a linearized buckling analysis, with very light formula due to the presence of displacement degrees of freedom only. Due to its efficiency it is suitable for layup design when geometrical nonlinearities have to be considered [51].

The paper is organized as follows: Section 2 presents the mixed solid-shell finite element for multi-layered composite structures; Section 3 derives the new Koiter asymptotic algorithm that exploits the third order only dependence of the strain energy on the FE parameters; Section 4 presents some numerical tests and discusses the accuracy of the proposed framework; finally the conclusions are reported in Section 5.

2. The mixed solid-shell finite element

In this section we briefly recall the mixed solid-shell finite element proposed by Sze et al. in [42] that is an effective extension of the initial PT18β hybrid element of Pian and Tong [52] to composite shell structures. The element is presented in a total Lagrangian formulation in order to be used in the Koiter strategy.

2.1. Kinematics in convective frame

We consider a solid finite element and denote with $\boldsymbol{\zeta} = \{\xi, \eta, \zeta\}$ the convective coordinates used to express the FE interpolation in natural coordinates. The initial configuration, assumed as reference, is described by the position vector $\mathbf{X}[\zeta] \equiv \{X[\zeta], Y[\zeta], Z[\zeta]\}$ while $\mathbf{x}[\zeta]$ represents the same position in the current configuration. They are related by the transformation

$$\mathbf{x}[\zeta] = \mathbf{X}[\zeta] + \mathbf{d}[\zeta] \tag{1}$$

where $\mathbf{d}[\zeta]$ is the displacement field. Adopting the convention of summing on repeated indexes, the covariant Green–Lagrange strain measure components are

$$\bar{\varepsilon}_{ij} = \frac{1}{2}\left(\mathbf{X}_{,i} \cdot \mathbf{d}_{,j} + \mathbf{d}_{,i} \cdot \mathbf{X}_{,j} + \mathbf{d}_{,i} \cdot \mathbf{d}_{,j}\right) \quad \text{with} \quad i,j = \xi, \eta, \zeta \tag{2}$$

where a comma followed by k denotes the derivative with respect to k and $(\cdot)$ denotes the scalar product. The position vector of a point inside the element and its displacement are interpolated, using a trilinear 8 node hexahedron, as