Composite Structures 150 (2016) 21–27

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

letter

Shear stiffness of neo-Hookean materials with spherical voids

Zaoyang Guo [a,b,c], Yang Chen [b], Xiongqi Peng [d], Xiaohao Shi [e,b], Haitao Li [b,*], Yuli Chen [c,*]

[a] State Key Laboratory of Coal Mine Disaster Dynamics and Control, Chongqing University, Chongqing 400044, China
[b] Department of Engineering Mechanics, Chongqing University, Chongqing 400044, China
[c] Institute of Solid Mechanics, Beihang University, Beijing 100191, China
[d] School of Materials Science and Engineering, Shanghai Jiao Tong University, Shanghai 200030, China
[e] Institute of Biomedical Engineering and Health Sciences, Changzhou University, Changzhou, Jiangsu 213164, China

ARTICLE INFO

Article history:
Received 22 December 2015
Revised 1 April 2016
Accepted 24 April 2016
Available online 25 April 2016

Keywords:
Particle-reinforced composite
Representative volume element
Neo-Hookean material
Numerical homogenization
Hyperelasticity
Porous materials

ABSTRACT

In this paper, the shear stiffness of neo-Hookean materials with spherical voids is investigated using a numerical homogenization approach. Three-dimensional representative volume element (RVE) models with non-overlapping identical randomly distributed spherical voids are generated to simulate the mechanical responses of the porous neo-Hookean materials. Pure shear deformations are simulated using the finite element method (FEM) and the numerical results show a clear linear relation between the nominal shear stress and the nominal shear strain. The effective shear stiffness of the porous neo-Hookean materials can then be computed by fitting the numerical data. It is further verified by more FEM simulations that the effective shear stiffness can be extended to predict the mechanical responses of the porous neo-Hookean materials subjected to any general isochoric deformations. It is also shown that the effective shear stiffness of the porous neo-Hookean materials can be well predicted as a function of the volume fraction of the voids using the classical three phase model originally proposed for linear elastic composites. Finally, RVE models with spherical voids of various sizes are created to study the effect of the size distribution of voids and the FEM results illustrate that it has negligible effect on the effective shear stiffness.

1. Introduction

Porous materials have been widely used in various engineering areas due to the low density and the capability to bear large deformation. In many applications they are subjected to shear deformations and the effective shear stiffness of porous materials is practically important. The effective mechanical properties of porous materials or composites in infinitesimal deformation region have been extensively investigated in the literature. Many classical models (e.g., [2–5]) can be employed to estimate the effective shear stiffness of porous linear elastic materials. Some rigorous bounds developed from variational principles (e.g., [6]) can also be applied to the effective properties of porous materials. These theoretical models were compared with the numerical homogenization results (e.g., [7]). However, the mechanical behavior of porous materials in the finite deformation regime is difficult to predict because of the combination of the intrinsic geometrical nonlinearity and material nonlinearity. Hill [8] proposed a framework to use macroscopic variables to describe the effective mechanical behavior of inhomogeneous materials in finite deformation. Hashin [9] investigated the mechanical response of porous hyperelastic materials under hydrostatic loading. Although some two-dimensional problems (e.g., composites with aligned fibers or cylindrical voids, see [10–15]) were solved analytically, it is difficult to get analytical approximations for three-dimensional composite models (e.g., particle-reinforced composites, see [16]). Nevertheless, Castaneda [17] developed a self-consistent model to estimate the shear stiffness of incompressible particle-reinforced neo-Hookean composites. Bergstrom and Boyce [18] estimated the shear modulus of rigid particle reinforced neo-Hookean composites using the concept of strain amplification. Danielsson et al. [19] proposed a micromechanics framework to derive the constitutive models for porous hyperelastic materials based on an approximate displacement field solution obtained by Hou and Abeyaratne [20]. However, the effective shear stiffness obtained from this model shows a linear relation with the volume fraction of the voids, which is incorrect for porous linear elastic materials. Lopez-Pamies and Castaneda [21,22] studied the mechanical behavior and instability of porous hyperelastic materials using the second-order estimate method.

* Corresponding authors.
E-mail addresses: htli@cqu.edu.cn (H. Li), yulichen@buaa.edu.cn (Y. Chen).

http://dx.doi.org/10.1016/j.compstruct.2016.04.046
0263-8223/

In this paper, we investigate the shear stiffness of neo-Hookean materials with spherical voids using a numerical homogenization approach. The porous material is simulated by the three-dimensional representative volume element [1] models with randomly distributed non-overlapping identical spherical voids. When the pure shear deformations are applied to the RVE samples, the numerical results obtained by the finite element method (FEM) show that the nominal shear stress is proportional to the nominal shear strain. We can then get the effective shear stiffness of the porous neo-Hookean materials by fitting the numerical data. After that, we apply general isochoric biaxial deformations, general triaxial shear deformations and general isochoric deformations to the RVE models. All the FEM results suggest that the effective shear stiffness obtained from pure shear deformations can be extended to predict the mechanical responses of the porous neo-Hookean materials subjected to any general isochoric deformations. We also find that the effective shear stiffness of the porous neo-Hookean materials can be well predicted by the classical three phase model (TPM) originally proposed for linear elastic composites [4] as a function of the volume fraction of the voids. Finally, we study the effect of the size distribution of voids by generating some RVE models with spherical voids of various sizes. The FEM results obtained from these RVE models illustrate that the size distribution of voids has negligible effect on the effective shear stiffness.

The structure of this paper is as follows: in Section 2, the theoretical basis of the numerical homogenization in finite deformation [8] is briefly introduced. In Section 3, after the RVE models are created, various deformations are applied to the RVE samples using FEM and the numerical results are presented. Then several issues are discussed in Section 4 and some concluding remarks are given in Section 5.

2. Theoretical basis of numerical homogenization in finite deformation

In this paper, we will investigate the shear stiffness of the neo-Hookean materials with spherical voids under finite deformation using a numerical homogenization approach. For a homogeneous isotropic hyperelastic material, its mechanical behavior can be characterized by a strain energy density function (per unit volume in its original configuration) $W = W(\mathbf{F})$, where the deformation gradient tensor $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X}$. Here $\mathbf{X}$ and $\mathbf{x}$ represent the positions of a material point respectively in the original (undeformed) and current (deformed) configurations of a solid. When the material is compressible, the nominal stress $\mathbf{P}$ is obtained as

$$\mathbf{P} = \frac{\partial W(\mathbf{F})}{\partial \mathbf{F}}, \quad P_{ij} = \frac{\partial W}{\partial F_{ij}}, \tag{1}$$

while for an incompressible hyperelastic material, $\mathbf{P}$ is computed as

$$\mathbf{P} = -p\mathbf{F}^{-T} + \frac{\partial W(\mathbf{F})}{\partial \mathbf{F}}, \tag{2}$$

where p is the (arbitrary) pressure. The widely used incompressible neo-Hookean model employs the following strain energy function:

$$W = \frac{\mu}{2}(I_1 - 3). \tag{3}$$

In this simple model, only one material parameter μ, which represents the initial shear modulus, is required. Here $I_1 = tr\mathbf{C}$ indicates the first invariant of right Cauchy-Green tensor $\mathbf{C} = \mathbf{F}^{\mathbf{T}}\mathbf{F}$.

For an inhomogeneous hyperelastic material (like the porous materials we study here), Hill [8] proposed a framework to define the constitutive macro-variables to represent the material's effective macroscopic mechanical behavior in finite deformation, which is briefly described as follows. Considering a representative volume of the inhomogeneous hyperelastic material, which occupies volume V in the undeformed configuration, the volume averages (denoted by an over-bar) of the deformation gradient $\mathbf{F}$, the nominal stress $\mathbf{P}$ and the strain energy W are computed as [8,23]

$$\overline{\mathbf{F}} = \frac{\int_{\mathbf{V}} \mathbf{F dV}}{\mathbf{V}}, \quad \overline{\mathbf{P}} = \frac{\int_{\mathbf{V}} \mathbf{P dV}}{\mathbf{V}}, \quad \overline{\mathbf{W}(\mathbf{F})} = \frac{\int_{\mathbf{V}} \mathbf{W}(\mathbf{F})\mathbf{dV}}{\mathbf{V}}. \tag{4}$$

Based on the equilibrium equations and the divergence theorem, the average deformation gradient $\overline{\mathbf{F}}$ can be derived as follows:

$$\overline{F_{ij}} = \frac{\int_S x_i n_j dS}{V}, \tag{5}$$

where S denotes the surface of the volume V and $\mathbf{n} = n_j\mathbf{e}_j$ is the outward unit vector normal to the surface S. This implies that the surface displacement can be employed to compute the average deformation gradient $\overline{\mathbf{F}}$. Similarly, for a continuum in equilibrium, the average nominal stress $\overline{\mathbf{P}}$ can be computed as

$$\overline{P_{ij}} = \frac{\int_S X_i P_{kj} n_k dS}{V}. \tag{6}$$

This suggests that the average nominal stress $\overline{\mathbf{P}}$ can be determined by the nominal stress $\mathbf{P}$ on the surface S. With these average macro-variables defined, Hill [8] showed that for compressible materials,

$$\overline{\mathbf{P}} = \frac{\partial \overline{\mathbf{W}(\mathbf{F})}}{\partial \overline{\mathbf{F}}}. \tag{7}$$

When the material is incompressible, it reads

$$\overline{\mathbf{P}} = -\mathbf{p}\overline{\mathbf{F}}^{-\mathbf{T}} + \frac{\partial \overline{\mathbf{W}(\mathbf{F})}}{\partial \overline{\mathbf{F}}}. \tag{8}$$

Here $\overline{W(\mathbf{F})} \equiv \overline{W}(\overline{\mathbf{F}})$ is treated as a function of $\overline{\mathbf{F}}$. Comparing with homogenous materials, $\overline{W}(\overline{\mathbf{F}})$ can be considered as the effective strain energy function which describes the macroscopic mechanical behavior of the overall composite. However, even for hyperelastic composites with very simple microstructures, because of the intrinsic material nonlinearity and geometrical nonlinearity, it is usually very difficult (if not impossible) to obtain an analytical expression for the effective strain energy function under a general deformation state [16,24].

An alternative approach is to use the numerical homogenization methods to investigate the effective behavior of hyperelastic composites [24–27]. Moraleda et al. [26] created two-dimensional RVE models to study the mechanical response of fiber-reinforced hyperelastic composites. Guo et al. [24] studied the mechanical behavior of incompressible particle-reinforced neo-Hookean composites numerically using three-dimensional RVE models.

In this paper, we investigate the shear stiffness of a neo-Hookean material with spherical voids. Here the voids are randomly distributed and the material's mechanical behavior is assumed isotropic macroscopically. Therefore, there are only two parameters (i.e., the shear modulus of the neo-Hookean material μ and the volume fraction of the voids c) in the model. Without losing generality, the shear modulus of the material can be set as $\mu = 1$, and we only need to consider one parameter c.

3. RVE models and finite element simulations

3.1. RVE models

A set of RVE models are generated to simulate the mechanical responses of neo-Hookean materials with non-overlapping spherical voids. The same microstructures of the RVE models adopted in Guo et al. [24] are employed while the spherical particles are replaced with voids. Therefore, 16 RVE samples are created for