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Sequential permutation table method for optimization of stacking sequence in composite laminates

Zhao Jing [a,b], Qin Sun [a,*], Vadim V. Silberschmidt [b]

[a] School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, China
[b] Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, LE11 3TU Leicestershire, UK

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ABSTRACT

This paper presents a new stacking-sequence optimization method for composite laminates using a multicriteria objective function with buckling, strength and continuity constraint subject to in-plane normal compressive loads. The objective function combines a critical buckling load factor function and a critical failure load function into a single function to maximizes in which the relative influence of a stacking sequence and number of ply orientations is reflected. The buckling-load and failure-load factors are sorted from maximum to minimum respectively in a sequential permutation table. A sequence-design strategy and a ply-orientation selection strategy are developed to identify the ply orientation of each stacking position and number of ply orientations of the stacking sequence involving continuity constraints based on the sequential permutation table. A benchmark problem is presented to illustrate the accuracy and effectiveness of the method; the detailed results are compared with various heuristic methods.

1. Introduction

Composite materials provide unmatched potential and large freedom in design to reduce their weight and/or to improve their performance. Thickness, a stacking sequence and ply orientations are often set as design variables to tailor global levels of stiffness and strength of composite laminates [1,2]. A discrete set of ply laminates makes it a non-convex, nonlinear and high-dimensional complex mixed-discrete optimization problem [3] with many local optima, thus bringing new challenges for the design and optimization process.

Though the stacking-sequence optimization is a mixed-discrete and high-dimensional optimization problem with discrete variables, modern mathematical optimization methods combined with specific techniques appears to be capable of providing efficient solution to this problem. Buckling is one of the most important constraints in composite structural optimization since it is one of the most common failure forms of composite structures [2,4]. A variety of optimization methods were applied to this problem [5,6]. The most popular method, namely a genetic algorithm (GA) [7–9], is used in stacking-sequence design with buckling, strength and continuity constraints. The problem, described in [7], of optimization of the buckling load factor by changing the ply orientations, attracts many researchers developing a variety of heuristic methods [8–20]. In particular, a permutation GA with a repair strategy was developed [12] for continuity constraints. Furthermore, a bi-level optimization scheme based on the permutation GA is proposed for the composite wing box design [14,15]. An integrated approach, combining a shape optimization process with the GA was proposed in [21] for shape optimization and stacking-sequence design of composite laminates. Another popular method is evolutionary algorithm (EA), used in multiobjective optimization of composite laminates [22] and tapered composite structures [23]. More recently, a permutation search (PS) is proposed in [24], where a buckling load factor was expressed as discrete forms of function of ply orientations to reduce computational cost.

On the other hand, optimization of multi-panel composite structures is more difficult due to blending of adjacent panels and nature of ply drops in design. The GA coupled with a response surface method was used in [18] to optimize both single-panel and two-patch design examples. In recent years, a stacking-sequence table (SST) scheme was specialized for EA-based blending optimization [23]. Recently, a global shared-layer blending (GSLB) [25] method was proposed based on a shared-layer blending (SLB) method [26] for blending design.

Although, various heuristic methods are applied to the stacking-sequence optimization problem, they are sensitive to initial values and predefined parameters. For specific optimization problems, the predefined parameters should be adjusted properly to find the optimal solutions. Unsuitable initial values or predefined

* Corresponding author.

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parameters may result in a low convergence rate or even fail to find the optimum. Additionally, the design space increases exponentially with the increasing number of design variables, making heuristic methods computationally expensive for solving optimization problems of large-scale composite structures. In order to overcome these limitations as well as concerns of convergence difficulties and of problem size of heuristic methods, a specific method is motivated by previous study for stacking-sequence optimization.

It is suggested that a stacking sequence of the laminate can be designed based on the classical laminate theory employing a specific designed algorithm [24,27,28]. In [27], a two-level method was developed to determine the feasible region of the lamination parameters. Recently, a bi-level optimization scheme for finding an optimal stacking sequence of composite laminates subjected to mechanical, blending and manufacturing constraints was developed in [28]. In this paper, based on a theory of lay-up optimization [27,28], a sequential permutation table (SPT) method was proposed for stacking-sequence optimization.

The remainder of this paper is arranged as follows: in Section 2, an optimization problem is formulated. Then, the SPT method is developed in Section 3. In Section 4, performance of the method is compared with several heuristic methods via a benchmark problem. Finally, some conclusions and suggestions are provided in Section 5.

2. Statement of optimization model

Buckling analysis of a symmetric and balanced composite plate simply supported on four edges (Fig. 1a), can be formulated using an analytical method [16]. The classical plate theory of composite laminate is briefly summarized in Appendix A. Considering an orthotropic plate, where $D_{16} = D_{26} = 0$, buckling of the plate into m and n half-waves (along the x and y directions) occurs when the load amplitude factor λ_b reaches the following value [16]:

$$\lambda_{cb} = \pi^2 \frac{\left[D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4\right]}{\left[F_x\left(\frac{m}{a}\right)^2 + F_y\left(\frac{n}{b}\right)^2\right]} \tag{1}$$

where F_x and F_y are the normal compressive loads applied to the plate in x and y directions, respectively. Substituting Eq. (A.1) into Eq. (1)

$$\lambda_{cb} = \sum_{k=1}^{N/2}(\lambda_{cb})_k = \frac{\pi^2}{F_x\left(\frac{m}{a}\right)^2 + F_y\left(\frac{n}{b}\right)^2}\sum_{k=1}^{N/2}\left[(D_{11})_k\left(\frac{m}{a}\right)^4 + 2((D_{12})_k + 2(D_{66})_k)\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + (D_{22})_k\left(\frac{n}{b}\right)^4\right] \tag{2}$$

In Eq. (2), λ_{cb} is expressed as the sum of $(\lambda_{cb})_k$, which is a linear function of the flexural stiffness parameters $(D_{ij})_k$. Obviously, for a specific stacking position k, the ply orientation θ_k is the only design variable in the optimization of flexural stiffness parameters and, thus, superposition principle is suitable for evaluation of the buckling load factor λ_{cb} [24]. As a result, the maximizing of λ_{cb} is equivalent to identification of the ply orientation θ_k at stacking position k. To simplify the optimization and design processes, the buckling load factor is formulated as a linear function of stacking sequence [24,28], therefore, the stacking sequence can be designed linearly, significantly reducing the computational cost.

Based on the aforementioned analysis, several important features of stacking-sequence design for the maximum buckling load can be summarized:

(I) At each stacking position k, the optimal ply orientation θ_k can be identified separately.
(II) For each part of the laminates, the stacking sequence can be formulated as independent variable. In other words, stacking sequence can be designed block by block.
(III) Compared to the plies near the midplane, the outermost plies make a major contribution to the buckling performance.

The above features imply that the sequence of the laminate can be designed linearly from the midplane to the outermost layer by selecting the proper ply orientation for maximizing the buckling load factor.

On the other hand, the failure load of the plate can be evaluated with the first-ply failure approach based on the maximum strain criterion [16]. Principal strains in the kth layer of the plate are related to the loads by the following relations

$$\begin{aligned} \lambda F_x &= A_{11}\varepsilon_x + A_{12}\varepsilon_y, \\ \lambda F_y &= A_{12}\varepsilon_x + A_{22}\varepsilon_y, \\ \lambda F_{xy} &= A_{66}\gamma_{xy}, \end{aligned} \tag{3}$$

where λ is the load factor, ε_x, ε_y and γ_{xy} are the global strain components. The strains for each orientation are given as

$$\begin{aligned} \varepsilon_1^k &= \cos^2\theta_k\varepsilon_x + \sin^2\theta_k\varepsilon_y + \cos\theta_k\sin\theta_k\gamma_{xy}, \\ \varepsilon_2^k &= \sin^2\theta_k\varepsilon_x + \cos^2\theta_k\varepsilon_y - \cos\theta_k\sin\theta_k\gamma_{xy}, \\ \gamma_{12}^k &= \sin 2\theta_k(\varepsilon_y - \varepsilon_x) + (\cos^2\theta_k - \sin^2\theta_k)\gamma_{xy}, \end{aligned} \tag{4}$$

where ε_1, ε_2, and γ_{12} are local strains of each orientation.

The strength failure load factor λ_{cf} is taken to be the largest load factor λ given by Eq. (3), which corresponds to the critical value when one of the local strains ε_1, ε_2, and γ_{12} of all layers reaches the ultimate allowable strain value: $\varepsilon_1^{ua} = 0.008$, $\varepsilon_2^{ua} = 0.029$ and

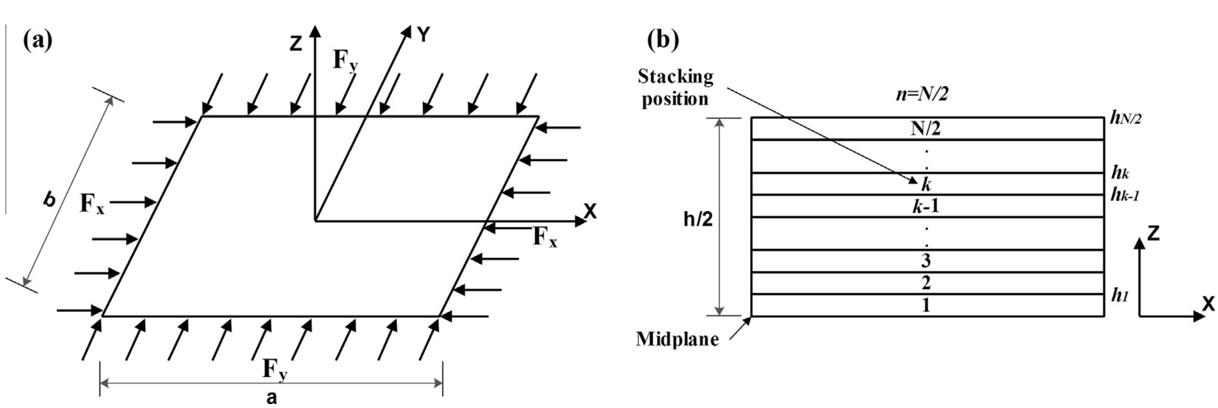

Fig. 1. (a) Loading and geometry of laminate; (b) stacking-sequence definition.