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Higher-order closed-form solution for the analysis of laminated composite and sandwich plates based on new shear deformation theories

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A B S T R A C T

In the present study, new shear deformation theories (algebraic (ADT), exponential (EDT), hyperbolic (HDT), logarithmic (LDT) and trigonometric (TDT)) were developed to analyze the static, buckling and free vibration responses of laminated composite and sandwich plates using Navier closed form solution technique. The present theories assume parabolic variation of transverse shear stresses through the depth of the plate. Besides, the transverse shear stresses vanish at the top and bottom of the plate surfaces. Thus, the necessity of shear correction factor is evaded. The governing differential equations and boundary conditions are obtained from the virtual work principle. Like FSDT, the present models consist of 5 unknowns. The shear stress parameter m that involves in shear strain function is selected through inverse method. To verify the accuracy and applicability of the present models, numerical comparisons were made with 3D elasticity solutions and existing theories. From the obtained results, it is observed that the proposed shear strain functions have significant effects on structural responses. Also, it is observed that the present theories are more accurate than the renowned theory, for the static, buckling and free vibration analysis of laminated plates.

1. Introduction

Composite materials have made gigantic strides over the past few decades in engineering fields such as aerospace, civil, naval, automotive and many other fields. The composite materials have distinguished characteristics such as high stiffness to weight ratio, high strength to weight ratio, outstanding fatigue strength and have the capability to tailor the lamination scheme according to the specific requirement. It possesses a low value of shear modulus than the homogenous isotropic plates. Consequently, it become much pronounced in the transverse shear deformation. Further, the existence of couplings among extension, shearing, bending, and torsion when the plate subjected to loading. In order to predict them effectively, development of an accurate mathematical model is necessary. The actual behavior of plates can be obtained through three-dimensional (3D) elasticity solution [1,2] at high computational cost and in this approach each lamina is taken as a complete 3D solid. Hence, the complexity of the theory intensifies with the increment of lamina. To overcome these effects, two-dimensional (2D) theories were developed that included equivalent single layer theories (ESL), Layerwise (LW) and Zigzag theories (ZZ) [3–8]. Among the single layer theories, Classical laminated plate theory (CLPT) [9] is the overlook of classical plate theory [10]. In which the transverse shear deformation is not considered and hence, the cross section perpendicular to the reference plane remains straight after deformation. Thus, CLPT limits its applicability to a high aspect ratio of plates and also, it underestimates the static response and overestimates the dynamic response. The first order shear deformation theory (FSDT) [11] is based on Reissner and Mindlin assumptions [12,13] which include the transverse shear effects with a linear variation of transverse shear strain through the plate thickness. Hence, to rectify the impractical behavior of the transverse shear strain/stress an artificial shear correction factor should be multiplied with the shear terms. Further, the shear correction factor dependent on layer orientation, loading conditions and boundary conditions.The FSDT theory widely adopted for laminated composite and FGM plates. Chen et al. [14] achieved nonlinear vibration results for FGM plate with Von Karman assumptions. Lanhe [15] given buckling results in thermal field for FGM plate. Panda et al. [16] studied the influence of hygrothermal effects on free vibration of delaminated plates.

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To overcome the limitations of CLPT and FSDT, several higher order shear deformation theories (HSDTs) were developed in the past few decades by assuming cubic, quadratic or higher variations of thickness coordinate. Basset [17] introduced the displacement field in terms of Taylor series expansion of thickness coordinate. Ambartsumian [18] presented bending results for plates and shells. Lo et al. [19] given a higher order plate theory including stretching effects in the thickness direction with 11 unknowns. Levinson [20], Murthy [21] and Reddy [22] developed the renowned theory. Putcha and Reddy [23] studied the dynamic effects on laminated plate with 11 unknowns. Kant et al. [24] presented a HSDT with 11 unknowns for the transient dynamic analysis using finite element methodology (FEM). Later, Kant and Swaminathan [25] given analytical solutions considering HSDTs with 12 and 9 unknowns. Pradyumna and Bandyopadhyay [26] studied static and dynamic responses of composite shells using higher order theory with 9 unknowns. Neves et al. [27] presented a plate theory with 9 unknowns for FGM plate where the in-plane displacement and transverse displacement components are assumed a cubic and quadratic order of thickness coordinate respectively. Talha and Singh [28] studied the static and vibration responses on functionally graded material (FGM) with 13 unknowns. Natarajan and Ganapathy [29] have presented FE results of a higher order plate theory with 13 unknowns for FGM plates. Though the above Taylor series expansion form of HSDTs yield to predict the structural responses with adequate accuracy, Yet they are physically quite complex to interpret in the formulation. Because, the Taylor series coefficient will generate extra unknown variables.

Several authors overcame the above mentioned complexity by developing shear deformation theories based on various shear strain functions. Levy [30], Stein [31] and Touratier [32] used a sinusoidal shear strain functions. Later on the same shear strain function have been handled by various authors for different theories [33–37]. Soldatos [38] proposed a sinusoidal hyperbolic shear strain function. Karama et al. [39] introduced an exponential function for the static and dynamic analysis of laminated plates. Afterwards, Aydogdu [40] redefined the work of Karama et al. [39]. Further, the same function again overlooked by Mantari et al. [41]. Akavaci [42] obtained analytical solutions for laminated plates by developing two hyperbolic functions. Meiche et al. [43] studied the dynamic analysis of FGM plates by proposing a hyperbolic trigonometric function. Mantari et al. [44,45] developed trigonometric shear deformation theories for plates and shells, afterwards the same theories have been assessed using zigzag theories [46,47]. Further, Mantari and Soares [48,49] have developed Quasi 3D plate theory with 6 unknowns for FGM plate. Neves et al. [50] studied a hyperbolic theory for FGM plate including transverse normal strain effects with 6 unknowns. Further, the same author's [51] have developed a sinusoidal hyperbolic theory for FGM plate with 9 unknowns by mesh free method. Mantari et al. [52] proposed a Quasi 3D trigonometric plate theory with 5 unknowns. Grover et al. [53] assessed the bending and buckling analysis of laminated plates using an inverse trigonometric function. Also, they [54] have examined the static and dynamic responses of laminated plate by proposing trigonometric functions. Thai et al. [55] have given vibration results for FGM plate using a Quasi 3D theory with 5 unknowns. Thai et al. [56] presented a inverse hyperbolic theory based on isogeometric method for laminated plates. Nguyen et al. [57] presented a trigonometric analytical solution for the advanced laminated plate with 5 unknowns. Similarly Belabed et al. [58] also presented a Quasi 3D plate theory for FGM plate using a trigonometric function of Mantari et al. [59] with 5 unknowns. Mantari et al. [60] proposed a quasi 3D plate theory with 4 unknowns for FGM plate. Al Khateeb and Zenkour [61] have developed a 4 unknown plate theory for FGM plate resting on elastic foundation in hygrothermal environment. Mohamed et al. [62] given a four unknown quasi 3D plate theory for FGM plate in hygrothermal environment. Zenkour [63] has given a 4-unknown Quasi 3D plate theory for composite plate resting on elastic foundation. Similarly several shear deformation theories were developed based on different shear strain functions with six unknowns [64,65], five unknowns [66–72] and four unknowns [73–76] for composite and FGM plates. Though the higher order theories have the capability to predict various structural behaviors accurately, still they are unable to account for transverse shear stress continuity at the layer interfaces. In order to include the shear stress continuity ZZ [77–79] and LW [80–84] theories are developed with high computational efforts. However, the continuity of shear stresses can be obtained by integrating the equilibrium equations [22] in HSDTs.

The best of author' s knowledge for the first time this paper presents higher-order closed- form solution for laminated composite and sandwich plates of static, buckling and free vibration analysis using new algebraic (ADT), exponential (EDT), hyperbolic (HDT), logarithmic (LDT) and trigonometric (TDT) shear deformation theories. Using the virtual work principle and calculus of variation the governing differential equations and boundary conditions are derived for the plate structure. A generalized formulation and coding are developed for shear deformation theory that includes shear strain function with five unknowns. The closed form Navier solution method is adopted to obtain the analytical solution for static and dynamic analysis. The shear stress parameter m which involves in shear strain function is optimized through inverse method. The present models represent non-linear variation of transverse shear stresses across the plate thickness. The transverse shear stresses are vanishes at the plate surfaces. Therefore, they neglect the shear correction coefficient. Also, the proposed models represent a non-linear variation of in-plane displacement across the plate thickness. The solution methodology is restricted to simply supported boundary conditions, nevertheless which doesn't carry computational and numerical error. Several numerical examples are conducted considering side to thickness ratio (a/h), length to width ratio (b/a), modulus ratio (E_1/E_2), core to face thickness ratio (t_c/t_f), number of layers (n), layer orientation (θ°) and loading conditions (q). The influences of the proposed shear strain functions on various structural behaviors are studied, and the evaluated results are validated with 3D elasticity solution and available different numerical techniques based shear deformation theories. To demonstrate the efficacy and accuracy of the developed theories global average error percentage with respect to 3D elasticity solution is done which made certain the applicability of the present models. The proposed theories have the potency to predict the structural responses of laminated composite and sandwich plates with adequate accuracy.

2. Theoretical formulation

Consider a rectangular laminated plate with N number of orthotropic layers as shown in Fig. 1. The plate is located in Cartesian coordinates $(x-y-z)$ with length a, width b and total thickness h. The fibers are oriented at θ° angle. The dotted line denotes the reference plane $(z=0)$. The proposed mathematical model can be represented in terms of generalized shear strain function $f(z)$ as

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - zw_{0,x} + f(z)\phi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) - zw_{0,y} + f(z)\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

where "$,x$" and "$,y$" are the partial derivatives with respect to x and y axis respectively. The in-plane displacement and transverse displacement components are denoted as u, v and w respectively. As well, the reference plane displacements are denoted as u_0, v_0 and