An efficient approach to the modeling of compressive transverse cracking in composite laminates

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ARTICLE INFO

ABSTRACT

A wedge-shaped failure mechanism occurs when a composite ply in a laminate is loaded under transverse compression. Therefore failure models developed for tension are not straightforwardly applicable to compression. For models that represent matrix cracks as discontinuities, the inclined crack topology complicates algorithms considerably. In this paper, a method is introduced to approximate the behavior of inclined cracks while avoiding these complications. Rather than adapting the crack topology, a transformation is applied in the evaluation of the cohesive law. The method is applied on predefined interface elements in a 2D model, as well as on cohesive segments in the extended finite element method in a 3D model. It is found that the compressive failure of composite laminates including the wedge effect can be approximated accurately with this method using a vertical crack plane. In 3D, a variable fracture plane angle is taken into account with the method, which means that the whole range of failure modes, from tensile through shear-dominated to compressive failure can be covered in a single approach.

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1. Introduction

During failure of composite structures, different failure processes may occur and interact. In composite laminates, the failure processes that are typically distinguished are fiber rupture, fiber kinking, matrix cracking, fiber–matrix debonding and delamination. Even though the individual processes occur on a small scale, a combination can eventually lead to ultimate failure of the structure. The complexity of the global failure process makes the development of reliable computational models for prediction of failure in composites both important and challenging. Much can be gained if reliable methods for the virtual testing of composites become available [1,2], but there are still challenges on different scales of observation.

A relevant scale of observation for composite failure is the mesoscale, where plies are modeled separately, but the microstructure of fibers and matrix material is not explicitly taken into account. A mesoscale model typically consists of a single layer of elements per ply or ply block. In this way, distinction between delamination and ply failure is easily made. Delamination can be properly modeled with interface elements (see e.g.[3–5]). To model ply failure, continuum damage mechanics methods have been developed that distinguish between fiber failure and inter-fiber

failure [6–8], also in compression [9]. However, Van der Meer and Sluys have shown that such models do not always lead to a correct representation of inter-fiber cracks [10]. Therefore, an approach where matrix cracks are represented as discrete discontinuities is to be preferred. This is possible with predefined interface elements [11–13], but with the high number of matrix cracks that may appear in a laminate, a mesh-independent representation with the extended finite element method (XFEM) is more suitable [14–19]. The XFEM, originally proposed by Moës et al. [20], allows for cracks to run through elements.

Inter-fiber failure can occur in different modes, depending on the loading. When a tensile transverse stress acts on the laminate, failure occurs with a fracture plane perpendicular to the mid plane of the ply. In compression load cases with a relatively high shear stress, shear dominated failure occurs where the fracture plane is still perpendicular to the mid plane of the ply. For compressive load cases with a relatively high compressive stress compared to the shear stress, compressive failure occurs where the fracture plane is inclined with respect to the mid plane of the ply. Puck and Schürmann [21] found that for pure compressive loading, the fracture plane tends to be inclined with respect to the plane perpendicular to the load with an angle of 53°. Dávila et al. [22] developed a set of ply failure criteria, where the critical crack plane angle for transverse compressive failure is determined iteratively depending on the compressive stress and shear stress in a plane stress situation. The dependence of the fracture angle on the stress

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state was confirmed experimentally in off-axis compressive tests on unidirectional specimens by Koerber et al. [23].

A generic model for predictive simulation of the complete failure process in laminates must account for the difference in crack topology of compressive failure compared to tensile and shear dominated failure. According to Puck and Schürmann [21], the inclined cracks can have an explosive effect because of their wedge shape which adds to the importance of including this geometric feature in failure models. However, recent models that make use of the XFEM for modeling matrix cracks [14–19] do not include a representation of inclined compressive cracks. So far, the assumption is made in these models that cracks are perpendicular to the ply mid-plane. This restricts the applicability of these models to tension or shear dominated load cases. Although it is possible to account for the angle of the fracture plane explicitly with the XFEM, the complexity of the algorithms that deal with the intersection of finite elements by the crack plane increases significantly when the crack plane is no longer perpendicular to the mid-plane of the ply.

In this paper, a method is presented that extends the applicability of the XFEM to transverse compressive failure while avoiding these complications. In order to achieve this, the compressive behavior of inclined cracks is modeled by applying a transformation of the cohesive law rather than providing an explicit representation of the crack topology. In Section 2, the method is introduced in 2D for pure compressive loading. It is applied to the cohesive law of interface elements that predefine the crack location and compared to a model where the inclined crack topology is incorporated. In Section 3, the method is applied to a 3D situation in the XFEM in which the fracture plane angle is made dependent on the stress state at the crack tip.

2. 2D Model

In this section, the method is described with which the behavior of compressive matrix cracks is approximated by applying a transformation of the cohesive law of interface elements perpendicular to the mid plane of the ply. For verification, the method is first applied to a unidirectional (UD) specimen loaded in transverse compression and compared to a model where the crack topology is predefined in the mesh with interface elements. Secondly, the effect of the transformation on modeling compressive matrix cracking followed by delamination is assessed with the analysis of a $[0/\overline{90}]_s$ laminate. A plane strain condition is considered throughout this section.

2.1. Interface Elements Formulation

Interface elements with the mixed mode damage law from Turon [4,24] serve as a basis for the transformed interface model. However, the transformation proposed below is applicable to any cohesive law. The cohesive law that relates the cohesive traction $\mathbf{t}$, applied to the crack surface, to the size of the displacement jump $\mathbf{j}$, is denoted

$$\mathbf{t} = \mathbf{t}(\mathbf{j}) \tag{1}$$

The cohesive law ensures that the interface elements initially approximate rigid behavior until a certain failure criterion is reached. Most, if not all, cohesive laws implement different behavior for normal deformations than for sliding deformations. This is done, among other reasons, because a negative displacement jump in normal direction should be prevented. In cohesive laws with damage, this is typically achieved by de-activating damage in normal direction in case of a negative displacement jump. In this way, the original high stiffness is recovered in normal direction when the crack faces are in contact.

In an interface element with $2m$ nodes, the displacement jump is defined as:

$$\mathbf{j} = \mathbf{Na} \tag{2}$$

With the shape function matrix

$$\mathbf{N} = \begin{bmatrix} N_1 & 0 & \dots & N_m & 0 & -N_1 & 0 & \dots & -N_m & 0 \\ 0 & N_1 & \dots & 0 & N_m & 0 & -N_1 & \dots & 0 & -N_m \end{bmatrix} \tag{3}$$

and

$$\mathbf{a}^{\mathrm{T}} = \{a_{1x}, a_{1y}, \dots, a_{2mx}, a_{2my}\} \tag{4}$$

where N_i is the shape function associated with node i and $\mathbf{a}_i^{\mathrm{T}}$ are the displacements of node i.

The displacement jump $\mathbf{j}$ in the global coordinate system is transformed to $\bar{\mathbf{j}}$ in the local $\{n, s\}$-frame before the cohesive law is evaluated in terms of local coordinates. For every integration point, this is done by rotation matrix $\mathbf{R}_\varphi$ that maps the global coordinates to the local coordinates. In 2D, it holds:

$$\bar{\mathbf{j}} = \begin{bmatrix} j_n \\ j_s \end{bmatrix} = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} j_x \\ j_y \end{bmatrix} = \mathbf{R}_\varphi \mathbf{j} \tag{5}$$

where φ is the angle between global x-axis and local n-axis. In the local coordinate system, the cohesive law (1) is applied to obtain the traction vector $\bar{\mathbf{t}}$ and its linearization with respect to the displacement jump $\bar{\mathbf{T}} = \delta\bar{\mathbf{t}}/\delta\bar{\mathbf{j}}$. The quantities $\bar{\mathbf{t}}$ and $\bar{\mathbf{T}}$ are transformed back into the global $\{x, y\}$-frame with the transposed rotation matrix $\mathbf{R}_\varphi^{\mathrm{T}}$:

$$\mathbf{t} = \mathbf{R}_\varphi^{\mathrm{T}} \bar{\mathbf{t}} \tag{6}$$

$$\mathbf{T} = \mathbf{R}_\varphi^{\mathrm{T}} \bar{\mathbf{T}} R_\varphi \tag{7}$$

The element force vector $\mathbf{f}_{\mathrm{elem}}$ and the element stiffness matrix $\mathbf{K}_{\mathrm{elem}}$ are then defined by an integral over the surface domain of the interface elements Γ_i:

$$\mathbf{f}_{\mathrm{elem}} = \int_{\Gamma_i} \mathbf{N}^{\mathrm{T}} \mathbf{t} \, \mathrm{d}\Gamma \tag{8}$$

$$\mathbf{K}_{\mathrm{elem}} = \int_{\Gamma_i} \mathbf{N}^{\mathrm{T}} \mathbf{T} \mathbf{N} \, \mathrm{d}\Gamma \tag{9}$$

2.2. Description

The method of applying a transformation to the cohesive law of an interface element, referred to as the 'transformed interface model', is explained in this section for a 2D situation. An elementary ply is considered, as shown in Fig. 2. The fiber orientation is perpendicular to the load direction, which means that the material is isotropic in the plane of the analysis. For pure compressive loading, the angle between the fracture plane and the plane perpendicular to the load, α, (see Fig. 2) can be considered to be a material property $\alpha_0 = 53°$ [21].

In order to approximate an inclined crack using vertical interface elements, the transformed interface model evaluates the cohesive law in the inclined fracture plane rather than in the plane aligned with the element. An additional coordinate frame is introduced. While the $\{n, s\}$-frame remains aligned with the interface element, the new $\{n^*, s^*\}$-frame follows the actual inclined crack plane. Instead of adjusting the angle of the original rotation matrix, an extra rotation is added to the model. In this way the transformation can be implemented as a wrapper around an existing cohesive law code. The transformation of the displacement jump $\bar{\mathbf{j}}$ from the local $\{n, s\}$-frame to $\bar{\mathbf{j}}^*$ in the inclined $\{n^*, s^*\}$-frame as shown in Fig. 1 is done by the rotation matrix $\mathbf{R}_\alpha$: