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Structure mechanical modeling of thin-walled closed-section composite beams, Part 1: Single-cell cross section

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ABSTRACT

An improved structure mechanical modeling with excellent accuracy is developed for single-cell thin-walled closed-section composite beams based on previous work. Both axial warping effect and the effect of material anisotropies on the shell wall mid-surface shear strain are considered. The shear strain is calculated directly from the general constitutive law of the shell wall. Closed form expressions are obtained of one-dimensional global beam stiffness matrix. Numerical comparisons with ABAQUS simulations are performed for box and cylindrical beams with a variety of lamina layups under various loading conditions and excellent agreements are observed. The effect of material anisotropies on the shell wall mid-surface shear strain has significant influence on the accuracy of modeling. In contrast, the axial warping effect has a negligible influence in cases considered. Significant deficiency of some existing models is revealed.

1. Introduction

The structures of thin-walled closed-section composite beams (TWCSCBs) have found wide applications in several engineering sectors. For example, modern large wind turbine blades are often made from fibre reinforced laminated composite materials, which are typically TWCSCBs as their span dimensions are much larger than their sectional dimensions. In practical optimisation design and analysis, the traditional one-dimensional (1-D) beam model is often preferred for efficient modelling of such structures made from these TWCSCBs benefiting from the reduced number of degrees of freedoms.

The 1-D modelling of thin-walled beams has attracted the attention of many researchers. Vlasov and Leontev [1] might be the first to develop such 1-D beam model with isotropic materials, followed by Gjelsvik [2] and Murray [3]. Their models are applicable to beams with both open and closed sections. The development of 1-D beam models with anisotropic materials was firstly reported by Mansfield and Sobey [4] and Mansfield [5]. However, their models do not account for shell wall bending and axial warping. Many research works have been conducted to improve the models [4,5]. It is worth noting that Kollar and Pluzsik [6] developed a 1-D beam model for single-cell composite beams with arbitrary layups and with both open and closed cross sections. In their model [6], the shell wall bending is considered; the effect of material anisotropies on the shell wall mid-surface shear strain in the case of closed sections is considered through two compatibility conditions enforced on an open cross section. However, their model [6] neglects the axial warping effect and results in a 4×4 1-D global beam stiffness matrix relating to global beam axial extension, two transverse bending and axial torsion. It is equally worth noting that the authors of the Ref. [6] extended their 1-D beam model [6] to include the transverse shear and axial warping effect in their new 1-D beam model [7]. However, the axial warping effect in their new model [7] is decoupled with the other four actions, i.e. axial extension, two bending actions and axial torsion, and is specifically calculated for a cantilever beam. As far as the authors' knowledge is concerned, Librescu and Song's work [8] contains the most detailed formulations for structure mechanical modelling of thin-walled composite beams. The work [8] is more recent and well known. With including the axial warping in consideration, the 1-D beam model in [8] results in a full 5×5 1-D global beam stiffness matrix relating to global beam axial extension, two transverse bending, axial torsion and bi-moment bending. However, the effect of material anisotropies on the shell wall mid-surface shear strain is not considered. Recent applications of the model [8] are found in the works by Lee and Lee [9], Vo and Lee [10–13], Cárdenas et al. [14], and many others.

The present work aims to develop an improved 1-D beam model for TWCSCBs with single-cell and multi-cell cross sections and with arbitrary lamina layups. Both axial warping and full material anisotropies are considered aiming particularly to demonstrate the effects of material anisotropies on the accuracy of the modelling.

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For the present work, neither cross-sectional transverse shear effect nor shell wall transverse shear effect is considered in order to focus on the effects of material anisotropies and axial warping. For the sake of a clear presentation and the length limit, the present work is reported in two continuous parts. This paper reports the part 1 for TWCSCBs with single-cell cross section and its sibling paper [15] reports the part 2 for TWCSCBs with multi-cell cross section.

2. Kinematic deformation

2.1. Global cylinder beam coordinate system OXYZ and local shell wall coordinate system nsz

A generic single-cell TWCSCB is shown in Fig. 1. Although kinematics for such TWCSCBs can be found in the work [8] of Librescu and Song and some text books, a complete presentation here is still required for the clear development of present model. To start with, the TWCSCB in Fig. 1 is considered to consist of a thin shell wall made from fibre reinforced laminated composite materials and a cylinder formed by the geometrical mid-surface of the shell wall. A global Cartesian coordinate system OXYZ is established for the cylinder and usually called global cylinder beam coordinate system or in short GCBCS. The *XY* plane is parallel to the cross-sectional plane of the cylinder and the directions of the axes *X* and *Y* are arbitrary. Of course, the *Z* axis is in the axial direction of the cylinder, but its position is also arbitrary. Also, a local orthogonal coordinate system *nsz* is established for the shell wall and usually called local shell wall coordinate system or in short LSWCS. The *ns* coordinate plane is parallel to the cross-sectional plane with *n* and *s* axes in the normal and tangent directions of the mid-surface of the shell wall, respectively. The *sz* coordinate plane is the mid-surface plane of the shell wall. Also, the *z* axis is in the axial direction of the cylinder. With these two coordinate systems the kinematics is developed below based on a series of geometric assumptions in addition to the small deformation assumption. Note that the fibre angle θ in Fig. 1 is measured relative to *s* and not the usual axial axis *z*.

2.2. Global cylinder beam displacements U(Z), V(Z), W(Z), Φ(Z) in GCBCS

Assumption (i) on the cylinder. The cross-sectional plane of the cylinder consisting of the empty space and the perimeter of the mid-surface of the shell wall experiences a rigid body movement under loads. In other words, the cross-sectional plane is rigid in its own plane.

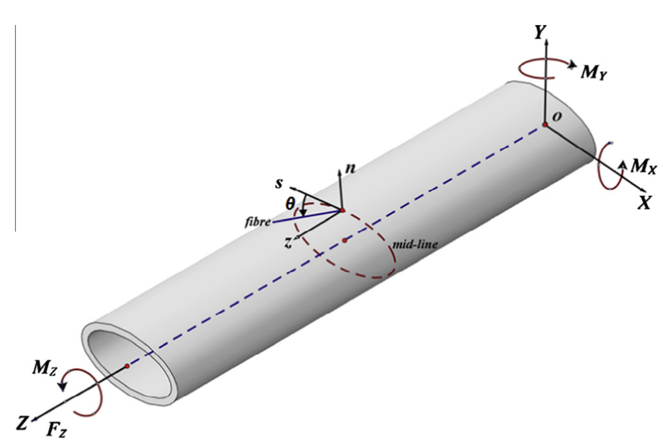

Fig. 1. Definitions of global cylinder beam coordinate system (GCBCS) and local shell wall coordinate system (LSWCS).

Note that this assumption assumes a rigid cross-sectional plane whilst the cylinder is of course deformable. The rigid body movement of a cross section at *Z* distance away from the origin of GCBCS consists of three translational displacements and three rotations. The three translational displacements can be represented by $U(0,0,Z) = U(Z)$, $V(0,0,Z) = V(Z)$, $W(0,0,Z) = W(Z)$ which are the displacements of a point $(0,0,Z)$ on the rigid cross-sectional plane as shown in Fig. 2. The three rotations can be represented by $\Phi_X(Z)$, $\Phi_Y(Z)$, $\Phi_Z(Z)$ which are rotations of the rigid plane about *X*, *Y*, *Z* axes, respectively, and positive when in positive *X*, *Y*, *Z* directions. These six rigid body displacements $U(Z)$, $V(Z)$, $W(Z)$, $\Phi_X(Z)$, $\Phi_Y(Z)$, $\Phi_Z(Z)$ represent the six fundamental displacements of the cylinder and are called global cylinder beam displacements for convenience in the present work. A major task in 1-D TWCSCB modeling is to determine them. Once they are known, the three translational displacements $U(X,Y,Z)$, $V(X,Y,Z)$, $W(X,Y,Z)$ of a generic point (X,Y,Z) on the rigid plane can be calculated.

$$U(X,Y,Z) = U(Z) - \Phi_Z(Z)Y \tag{1a}$$

$$V(X,Y,Z) = V(Z) + \Phi_Z(Z)X \tag{1b}$$

$$W(X,Y,Z) = W(Z) + \Phi_X(Z)Y - \Phi_Y(Z)X \tag{1c}$$

Assumption (ii) on the cylinder. The rigid cross-sectional plane after rigid body movement in Assumption (i) remains perpendicular to its deformed axial axis.

This assumption leads to

$$U'(Z) = \Phi_Y(Z), V'(Z) = -\Phi_X(Z) \tag{2}$$

where the prime superscript represents the ordinary differentiation with respect to *Z*. Eq. (2) indicates the transverse shearing effects on the deformation of the cylinder are neglected. This assumption reduces the six fundamental global cylinder beam displacements $U(Z)$, $V(Z)$, $W(Z)$, $\Phi_X(Z)$, $\Phi_Y(Z)$, $\Phi_Z(Z)$ to four, i.e. $U(Z)$, $V(Z)$, $W(Z)$, $\Phi(Z) = \Phi_Z(Z)$ as shown in Fig. 2. The aim of 1-D modeling of non-globally shear deformable TWCSCBs is to develop a mechanical model based on these four fundamental global cylinder beam displacements.

2.3. Local shell wall mid-surface displacements $\bar{u}(s,z)$, $\bar{v}(s,z)$, $\bar{w}(s,z)$ in LSWCS

Assumption (iii) on the perimeter of the mid-surface of the shell wall. Non-rigid warping in the axial direction is allowed in addition to the rigid body movement.

Using this assumption and Eqs. (1) and (2), the displacements of a generic point $(X(s), Y(s), Z)$ on the perimeter can be expressed as

$$U(X(s),Y(s),Z) = U(Z) - \Phi(Z)Y(s) \tag{3a}$$

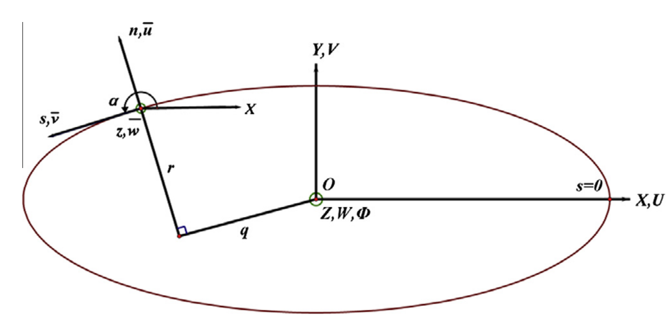

Fig. 2. Notations for sectional coordinates and displacements.