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# Crushing characteristics of fiber reinforced conical tubes with foam-filler

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ABSTRACT

In this paper, crushing force and energy absorption of foam-filled conical tubes with fiber reinforced layer between two metal walls under axial loading have been studied by means of an analytical method. Based on the axially crushing models, a simplified analytical solution for the static crushing of foam-filled fiber reinforced conical tubes is presented. The influences of fiber reinforced orientation, fiber layer thickness and base angle of conical tube on energy absorption capability were studied in examples. A validated finite element method was introduced to simulate the collapse of foam filled metal conical tubes and indirectly verified the feasibility of the simplified analytical model proposed in this paper.

## 1. Introduction

Crashworthiness of composite structures is an extensive research field for designing protective structures of automotive and aerospace fields, which provide significant capability of energy dissipation because of their higher strength-to-weight ratio, to avoid potential injury to passengers and cargo [1].

Thin-walled tubes are considered to be desirable energy dissipation systems. Those energy absorption structures have various cross-section shapes (i.e. circular tubes, square tubes, conical tubes) and are strengthened by composite material such as fiber-reinforced layer and aluminum foam. The energy absorption capability is closely related to dimensions of structures, constraint conditions, material properties and so on.

Based on the simplified model for axisymmetric fold pattern of thin-walled circular tubes [2], external work done is equal to internal energy from circumferential stretching and bending at stationary plastic hinges that were consider to move completely outwards. Singace et al. [3] proposed a new model which allowed outward and inward radial movement of plastic hinges according to a geometric eccentricity factor $m$. Later, the crashworthy behaviors of thin-walled tubes of fiber composite materials subjected to axial loading were studied in Refs. [4–8]. Based on the concertina collapse modes observed from experimental tests of reinforced metal tubes, a simplified analytical mechanism for the externally reinforced metal tubes was presented by Hanefi and Wierzbicki [9]. The mean crushing force calculated from this simplified mode was found to be in good agreement with experimental data.

Seitzberger et al. [10] revealed that the aluminum foam as filler material in tubes significantly improved energy absorption capability of thin-walled metal tubes due to the deformation of aluminum foam and the interaction between aluminum foam and metal wall, the most important is that low densities foam can preserve the progressive buckling modes which lead to higher energy absorption. Hanssen et al. [11] presented a formula to predict the mean crushing force of foam-filled tubes based on the experimental data. Kavi et al. [12] also used the three plastic hinge models to analyze the energy absorption and the mean crushing force of foam-filled circular tubes, the predicted values were found to be good agreement with experimental data.

Conical thin-walled tubes, as considerable crashworthy components, have received a great deal of attentions. It was observed from the experiments conducted by Mamalis et al. [13] that the initial collapsed modes of empty conical tubes were axisymmetric folds, as collapse going on, the initial axisymmetric folds changes into non-symmetrical diamond shape. Alghamdi et al. [14], Sobky et al. [15] and Singace et al. [16] revealed that the constraint conditions had a significant effect on the energy absorption of conical tubes. Mamalis et al. [17] introduced the explicit code LS-DYNA to simulate the crushing behavior of thin-walled metal circular tubes, which was found that the simulations conformed with the test result well.

Ahmad and Thambiratnam [18,19] used the validated explicit finite element code to investigated the impact response of foam-filled conical tubes subjected to quasi-static and dynamic loads. The simulated collapses of both empty and foam-filled tubes were verified to well conform to the tests. Ahmad and Thambiratnam

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### Nomenclature

| | |
|---|---|
| $\alpha$ | base angle of conical tubes |
| $t$ | wall thickness of conical tubes |
| $t_m$ and $t_c$ | thickness of metal walls and thickness of fiber reinforced layers |
| $H$ | length of the first fold |
| $\lambda$ | proportionality coefficient of adjacent folds |
| $m$ | geometric eccentricity factor |
| $W_w$ | total dissipation energy due to the collapse of fiber reinforced tubes wall |
| $F_m$ | mean crushing force |
| $V_{f0}$ | initial volume of the collapse foam filler |
| $V_{f1}$ | compression volume of foam filler |
| $W_f$ | energy absorbed by the compression of foam filler |
| $\sigma_{pl}$ | plateau stress of foam filler |
| $F_m$ | average resistance provided by foam filler |
| $F_{\text{int}}$ | axially crushing force provided by wall-foam interaction |
| $\sigma_L$ and $\sigma_T$ | stresses along the reinforced material's main directions $L$ (parallel to fibers) and $T$ (perpendicular to fiber) |
| $\tau_{LT}$ | shear stress in $L-T$ plane |
| $\sigma_{ft}$ and $\sigma_{fx}$ | stresses along the circumferential direction and axial direction of fiber layer |
| $\varphi$ | angle of fiber reinforced orientation |
| $\rho_f$ and $\rho_s$ | density of aluminum foam and density of solid cell of the aluminum foam |

[18] demonstrated the feasibility and superior performance of foam-filled tubes as energy absorption components, the presence of the foam filler in the conical tubes can improve crush stability and preserve the progressive crushing modes which were also observed in this paper.

According to different views, approaches frequently used to study the crushing characteristic and energy absorption of structures are mainly the experimental, numerical calculation and analytical methods. The crushing modes and energy absorption of structures are directly observed and understudied by using experimental approaches [9,12,13]. Based on numerical simulation methods [17–19], the crushing characteristics of complex structures considering differently constrained ends and loads can be simulated. However, these experimental and numerical results are limited to fixed experimental conditions and calculation parameters. It is difficult to describe the generalizations of crushing characteristics of structures or to enhance the energy absorption capability of structures by optimizing the parameters of structures due to the complexity of the results in the individual situations. Therefore, it is significant to develop some analytical solutions to predict the crushing force and energy absorption capability of structures based on different crushing model [2,3,9,20]. Due to the complexity to investigate the energy absorption characteristics of foam-filled conical tubes with fiber reinforced layers, the analytical method to predict the crushing force and energy absorption capability of the foam-filled conical tubes with a fiber reinforced layer is not presented in literatures so far.

This paper introduces a simplified analytical model for FFC (foam-filled conical) tubes with fiber reinforced layers to predict the mean crushing force and energy absorption. The presence of the foam filler makes conical tubes collapse in the progressive concertina folds which have higher energy absorption capability. We assume that the fiber reinforced FFC tubes collapse in a progressive concertina mode which is shown in Fig. 1. The theoretical prediction of foam-filled metal conical tubes was found to good agreement with the simulated results of validated explicit finite element method. By optimizing the geometrical and material parameters of the fiber reinforced FFC (foam-filled conical) tubes based on the present analytical method, the fiber reinforced FFC tubes with the maximum energy absorption capability can be designed.

## 2. Analytical method and solving process

Based on the simplified crushing modes for cylindrical metal shells [2] and the analytical method for fiber reinforced layers proposed by Wang and Lu [20], an analytical mode for energy absorption of fiber reinforced FFC tubes is proposed, as shown in Fig. 1, where the internal hinges (hinge 1, 3, 5 and so on) have the same inward radial displacement, the length of the first fold is $H$, the length of the second fold is $\lambda H$ and so on. $\lambda$ is the proportionality coefficient of adjacent folds length and $m$ is the geometric eccentricity factor. According to this analytical mode, the external energy is dissipated by three mechanisms: (a) progressive collapse of fiber-reinforced wall, (b) compression of foam filler, (c) interaction between foam filler and fiber-reinforced wall.

We focus on the analysis of energy dissipation of the first two collapse folds, and it will be easy to determine the energy dissipation of the subsequent two collapse folds by replacing $H$ and $r_0$ with $\lambda^2 H$ and $r_0 + 2(1+\lambda)H\cos\alpha$, respectively in the formulas.

### 2.1. Energy dissipation due to collapse of fiber-reinforced wall

The derivation processes of the energy dissipation induced by the sandwich wall of conical tubes are stated at Appendix A for the sake of brevity. The total dissipation energy due to the collapse of fiber reinforced tubes wall is

$$W_w = B_1 H^2 + B_2 H + B_3 \tag{1}$$

where

$$B_1 = 4(1+\lambda)^2 A_1 \cos^2\alpha + A_2, \quad B_3 = A_1 r^2 + \pi A_3 r$$

$$B_2 = 4(1+\lambda) r A_1 \cos\alpha + 2(\pi-2)(1+\lambda) A_3 \cos\alpha + (3+\lambda) A_3 \cos\alpha + A_4$$

$$A_1 = \frac{4}{3}\pi t_c \sigma_{ct} \varepsilon_{ct}^2, \quad A_2 = 4\pi (C_2 \cos\alpha + 2C_3 \sin\alpha),$$

$$A_3 = \frac{4}{\sqrt{3}}\pi \sigma_0 t_m^2 C_0, \quad A_4 = \frac{4}{\sqrt{3}}\pi \sigma_0 t_m^2 C_1 \sin\alpha$$

$$\begin{aligned} C_2 = {} & 2[\lambda^2 + (2m-1)\lambda - (1-m)(1+3\lambda)]\sigma_0 t_m \\ & - (1-m)(1+3\lambda)\sigma_{cc} t_c \end{aligned}$$

$$C_3 = [m^2 + (\lambda+m-1)^2 + 2(1-m)^2]\sigma_0 t_m + (1-m)^2 \sigma_{cc} t_c \tag{2}$$

The predicted formulation for the mean crushing force $F_m$ provided by tubes wall is

$$F_m = \frac{W_w}{[1.72(1+\lambda)H - 2t]\sin\alpha} = \frac{B_1 H^2 + B_2 H + B_3}{[1.72(1+\lambda)H - 2t]\sin\alpha} \tag{3}$$

where

$$H = \frac{4t + \sqrt{16t^2 + 6.88(1+\lambda)[1.72(1+\lambda)B_3 + 2B_2 t]/B_1}}{3.44(1+\lambda)} \tag{4}$$