Composite Structures 116 (2014) 404–413

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Dynamic eigenstrain behavior of magnetoelastic functionally graded cellular cylinders

A.H. Akbarzadeh [a,b,*], J.W. Fu [a,c], Z.T. Chen [a,*], L.F. Qian [c]

[a] Department of Mechanical Engineering, University of New Brunswick, Fredericton, NB E3B 5A3, Canada
[b] Department of Mechanical Engineering, McGill University, Montreal, QC H3A 0C3, Canada
[c] Department of Mechanical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China

ARTICLE INFO

Article history:
Available online 6 June 2014

Keywords:
Cellular material
Dynamic eigenstrain
Functionally graded cellular material
Magnetoelasticity
Multilayered cylinder
Multiscale modeling

ABSTRACT

The magnetoelastic behavior of heterogeneous thick-walled cylinder with cellular material layers and enduring a dynamic and spatially varying eigenstrain is studied in this paper. The electrically conducting cylinders under plane strain or plane stress condition are subjected to a constant magnetic field. An efficient methodology is developed for the time-harmonic and transient response of multilayered cylinders. The developed methodology is then employed to model the functionally graded cellular cylinders with an arbitrary profile of the relative density distribution via the piecewise homogeneous layer technique. The results are first verified with those available in literature. Then, the effect of relative density, non-homogeneity index, and geometric configuration is examined on the dynamic magnetoelastic response of cylinders containing cellular material layers.

1. Introduction

Multilayered and laminated composite have received increasing attention in aerospace, automobile, transportation, piping, and biomedical industries [1,2]. The conventional laminated composites are prone to delamination, deboning, and crack initiation due to the mismatch of material properties of bonded layers [3]. These are the reason for which functionally graded materials (FGMs) with continuous variation of material properties have been introduced [4]. This gradual variation in material properties reduces the likelihood of fracture caused by the stress concentration, in-plane and out-of-plane thermal stresses, and high stress intensity factors in composite laminates [5,6]. As a result, extensive research has been devoted to the multiscale modeling of multilayered and FGM structures.

The microscopic and macroscopic modeling of composite laminates has been the research topic in several studies [7,8]. For instance, a failure analysis was conducted by Hakki Akcay and Kaynak [9] for multilayered cylinders subjected to thermomechanical loading. Tsukrov and Drach [10] provided explicit expressions for displacement and stress fields in a multilayered cylinder with orthotropic layers. Kuo [11] also examined the behavior of smart circular fibrous composites with imperfect interfaces.

Furthermore, the structural behaviors of functionally graded (FG) structures have drawn the attention of researchers in the last two decades [12,13]. For example, the elastic analysis of a thick FG cylinder with the exponential material profile was conducted by Chen and Lin [14]. Akbarzadeh et al. [15–17] employed the hybrid Laplace–Fourier transform to study the transient thermomechanical behavior of FG plates and doubly-curved panels.

In micromechanics, an eigenstrain could simulate several multiphysics phenomena, such as a plastic deformation, hygrothermal strain, and misfit strain. The self-equilibrated stress caused by the eigenstrain is called eigenstress [18,19]. The eigenstrain and inclusion analyses have been used by many scientists in order to elucidate the behavior of multiphase and heterogeneous composites [20,21]. Two methodologies using the eigenstrain model were presented by DeWald and Hill [22] and Achintha et al. [23] to predict the residual stresses within the multi-dimensional media. Bromley et al. [24] also introduced a model via eigenstrain analysis to approximate the residual stresses caused by the thermally induced plasticity. In addition, electrically conducting composites, working in the presence of magnetic field, experience the Lorentz force based on the magnetoelasticity theory [25,26]. Since the external magnetic field changes the multiphysics behavior of smart composites, micromechanical approach as well as multiphysics have been employed in references [27–29] to obtain the constitutive

* Corresponding authors at: Department of Mechanical Engineering, University of New Brunswick, Fredericton, NB E3B 5A3, Canada. Tel.: +1 (506) 458 7784; fax: +1 (506) 453 5025.

E-mail addresses: h.akbarzadeh@unb.ca, hamid.akbarzadeh@mail.mcgill.ca (A.H. Akbarzadeh), ztchen@unb.ca (Z.T. Chen).

http://dx.doi.org/10.1016/j.compstruct.2014.05.036
0263-8223/

models for magnetoelastic composites. Some closed-form solutions were also achieved in references [30,31] for dynamic analysis of magnetoelastic media.

While most eigenstrain and magnetoelastic analyses deal with solid materials, there exist only a few contributions on the eigenstrain problems in media composed of cellular materials. Cellular materials with distinctive multiphysics and multifunctional properties offer a robust low-mass solution for applications that require light-weight and stiff structural components [32]. Lattice materials are periodic cellular structures whose material properties are controlled by cell geometry, nodal connectivity, and relative density [33,34]. Due to the controllability of material properties of cellular structures, they have found many applications in aerospace, biomedical, and architecture industries. The effective electromagnetic properties of lattices were found in references [35,36] using the numerical and experimental investigations. The plane wave propagation in infinite two-dimensional periodic lattices was also studied theoretically by Phani et al. [37]. Pasini et al. [38–40] conducted the multiscale modeling of heterogeneous, hierarchical, and multi-dimensional lattices using the homogenization techniques.

Due to the application of sandwich structures, composed of cellular layers, under prescribed multiphysics loadings, this paper examines the magnetoelastic behavior of multilayered and functionally graded cellular cylinders subjected to a dynamic eigenstrain. The dynamic eigenstrain could be interpreted as a sudden thermal expansion in the internal layer of a bi-layered tube due to a sudden or periodic temperature rise. This phenomenon can occur in the multilayered tubes containing hot water or wheels at braking. A methodology is developed for the eigenstrain analysis in multilayered cylinders with perfectly bonded interfaces which is employed for studying the eigenstrain behavior of functionally graded structures. Using the multiscale modeling, the homogenized material properties of cellular materials with square cell topology are used to study the dynamic responses of heterogenous cylindrical tubes/disks composed of cellular materials under the dynamic eigenstrain. Eventually, maps are given to clarify the influence of relative density, non-homogeneity index, and geometrical configuration on the magnetoelastic responses.

2. Problem definition and governing equations

An electrically conducting, multilayered, cylindrical tube (plane strain) or disk (plane stress) working in the presence of external magnetic field, H_z, endures a dynamic eigenstrain in arbitrary layers. As illustrated in Fig. 1, R_0 is the inner radius of the first layer, while the outer radius of the kth layer ($k = 1,2,\ldots,N$) is represented by R_k; the number of layers is also represented by N.

Suppose the kth layer of the cylindrical tube/disk undergoes a dynamic eigenstrain with a cubic polynomial radial distribution [30]:

$$\varepsilon_{mn}^{*(K)} = \delta_{mn} E^{*(k)}(r)\varphi(t) \tag{1}$$

where the superscript k denotes the kth layer of the cylindrical tube/disk; δ_{mn}(m, $n = r$, θ, z for cylindrical coordinate), r, and t are, respectively, Kronecker delta function, radial coordinate, and time. The cubic polynomial radial distribution of eigenstrain with arbitrary coefficients $A_l^{(K)} (l = 0, 1, 2, 3)$ is $E^{*(K)}(r) = A_0^{(K)} + A_1^{(K)} r + A_2^{(K)} r^2 + A_3^{(K)} r^3$; φ also stands for the time dependence function of the eigenstrain.

The elastic strain $\varepsilon_{mn}^{(K)}$ relates eigenstrain $\varepsilon_{mn}^{*(K)}$ and total strain $e_{mn}^{(K)}$ as [18]:

$$e_{mn}^{(K)} = \varepsilon_{mn}^{(K)} + \varepsilon_{mn}^{*(K)} \tag{2}$$

The strain components as a function of radial displacement $u^{(K)}$, for the considered axisymmetric problem, are [41]:

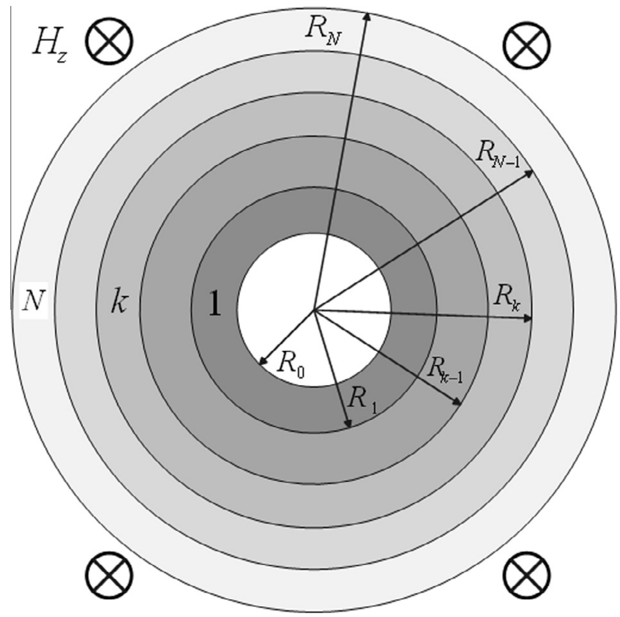

Fig. 1. A multilayered cylindrical tube/disk in a magnetic field.

$$e_{rr}^{(k)} = u_{,r}^{(k)}, \quad e_{\theta\theta}^{(k)} = \frac{u^{(k)}}{r} \tag{3}$$

where the subscript comma represents the differentiation with respect to the radial coordinate. The constitutive equation also reads:

$$\sigma_{mn}^{(k)} = 2G'^{(k)}\left(\varepsilon_{mn}^{(k)} + \frac{\nu^{(k)}}{1-2\nu^{(k)}}\varepsilon^{(k)}\delta_{mn}\right) \tag{4}$$

where σ_{mn}, ν, and $\varepsilon = \varepsilon_{mm}$ represent stress components, Poisson's ratio, and volumetric strain based on the Einstein summation convention; $G' = \frac{E}{2(1+\nu)}$, and E denotes Young's modulus. Although G' is the same as shear modulus G for isotropic materials ($G' = G$); $G' \neq G$ for cellular materials with cubic symmetry. Substituting Eqs. (1)–(3) into Eq. (4) yields the following stress fields for plane strain and plane stress conditions [42,43]:

$$\sigma_{rr}^{(K)} = \begin{cases} \frac{2G'^{(K)}}{1-2\nu^{(K)}}\left[(1-\nu^{(K)})u_{,r}^{(K)} + \nu^{(K)}\frac{u^{(K)}}{r} - (1+\nu^{(K)})E^{*(K)}(r)\varphi(t)\right], \\ \quad \text{plane strain} \\ \frac{2G'^{(K)}}{1-\nu^{(K)}}\left[u_{,r}^{(K)} + \nu^{(K)}\frac{u^{(K)}}{r} - (1+\nu^{(K)})E^{*(K)}(r)\varphi(t)\right], \\ \quad \text{plane stress} \end{cases} \tag{5a}$$

$$\sigma_{\theta\theta}^{(K)} = \begin{cases} \frac{2G'^{(K)}}{1-2\nu^{(K)}}\left[\nu^{(K)}u_{,r}^{(K)} + (1-\nu^{(K)})\frac{u^{(K)}}{r} - (1+\nu^{(K)})E^{*(K)}(r)\varphi(t)\right], \\ \quad \text{plane strain} \\ \frac{2G'^{(K)}}{1-\nu^{(K)}}\left[\nu^{(K)}u_{,r}^{(K)} + \frac{u^{(K)}}{r} - (1+\nu^{(K)})E^{*(K)}(r)\varphi(t)\right], \\ \quad \text{plane stress} \end{cases} \tag{5b}$$

$$\sigma_{zz}^{(K)} = \begin{cases} \frac{2G'^{(K)}\nu^{(K)}}{1-2\nu^{(K)}}\left[u_{,r}^{(K)} + \frac{u^{(K)}}{r} - \frac{1+\nu^{(K)}}{\nu^{(K)}}E^{*(K)}(r)\varphi(t)\right], & \text{plane strain} \\ 0, & \text{plane stress} \end{cases} \tag{5c}$$

The equation of motion for an electrically conducting cylinder in the presence of magnetic field is [26]:

$$\sigma_{rr,r}^{(K)} + \frac{\sigma_{rr}^{(K)} - \sigma_{\theta\theta}^{(K)}}{r} + f_z^{(K)} = \rho^{(K)}\ddot{u}^{(K)} \tag{6}$$