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A new technique to optimize the use of mode shape derivatives to localize damage in laminated composite plates

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ABSTRACT

Damage localization in laminated composite structures is a very active area of research due to the role that these kind of structures play in the transport industries. The mode shape derivatives, like rotations (first derivative), curvatures (second derivative) and, more recently, third and four derivatives, have been used to localize damage in composite plates. The most used method to compute these derivatives is the application of finite differences. However, finite differences present several well-known problems, such as the error propagation and amplification. The magnitude of the error associated with the computed derivative is not easy to estimate, mainly because the numerical error associated with finite differences depends on the values of derivatives of higher order than the order of the derivative that one wants to compute. A new technique based on the Ritz method to estimate this error is proposed in this paper. The optimal spatial sampling for the numerical differentiation of the mode shapes are defined based on the minimization of the total error. The good performance of the optimal sampling is shown by applying it to the damage localization in a laminated composite plate.

1. Introduction

The use of composite materials, namely carbon fiber reinforced polymers, in aeronautical and aerospace structures is increasing due to the opportunities they present for weight reduction. In addition to their high specific stiffness and strength, other advantages include their superior fatigue performance. In spite of these advantages, composite materials are more sensitive to certain type of damages and present different kinds of defects or damage mechanisms from those of metals [1]. One of the most common damages is delamination, which usually cannot be detected by visual means. Therefore, non-destructive testing (NDT) techniques are of critical importance for structural integrity evaluation and failure prevention of engineering components made of composite materials. There are many NDT techniques to monitor damage, including X-ray, acoustic emission and ultrasonic methods. However, in most cases they operate locally and/or require special sample preparation and the removing of the inspection parts, leading to time consuming and increasing the maintenance costs. An alternative to the use of this kind of techniques are methods based on vibration characteristics, which can be associated with a broader structural health monitoring (SHM) technique. These methods allow global inspection and usually do not require special sample preparation and the removing of the parts to inspect. Several of these methods have been reviewed and surveyed over the years (see e.g. [2–7]). The reviews by Montalvao et al. [6] and Zou et al. [7] are of particular relevance since they reference vibration based methods applied to composite structures.

Among these methods, the differences of mode shape curvatures of undamaged and damaged structures was initially proposed by Pandey et al. and applied to beams [8]. More recently, this method has been applied to damage localization on composite plates [9,10]. According to Abdo and Hori [11], the differences in the rotation of mode shapes, i.e. the first derivative of modal displacement fields, can also be used to localize damage, namely in beams and plates. Besides the use of differences in first and second order derivatives of the modal displacement fields, one can also use differences in higher order derivatives to localize damage, like third and fourth order derivatives, namely in beams [12–15] and plates [16,17]. The derivatives are usually computed by applying finite differences. For instance, the mode shape curvatures can be computed by applying the second order central finite difference formula to the modal displacement fields. However, due to the approximative nature of finite differences, they lead to propagation and amplification of the measurement errors and noise which are

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always present in experimental data. In order to minimize this problem, one needs to select an optimal spatial sampling, since mode shapes undersampling and oversampling can have adverse effects on the quality of the damage localization [18]. Unfortunately, the magnitude of the error in the computation of the derivative is not easy to estimate, mainly because the numerical error associated with finite differences depends on the values of derivatives of higher order than the order of the derivative that one wants to compute.

A new technique based on the Ritz method to estimate the error in the computation of mode shape derivatives is proposed in this paper. Once the total error is estimated, the optimal spatial sampling used in the finite differences formulas to minimize the influence of the error can be defined. First, second, third and fourth order derivatives are computed using finite differences with a quadratic convergence of the spatial sampling, in order to localize damage in a laminated composite plate. It is observed that the optimal spatial sampling depends on the mode shape used and the accuracy of the measured data. It also depends on the order of the derivative and the kind of finite difference, namely the number of points used and the relation between the point where the derivative is computed and neighboring points. It is also shown in this paper that by using a spatial sampling close to the optimal value, one is able to obtain good damage localizations.

2. Method

2.1. Ritz method for orthotropic laminated composite plates

Considering the Kirchhoff assumptions [19], the maximum strain energy of a rectangular orthotropic plate with an in-plane area A is defined by [17]:

$$U_{\max} = \frac{1}{2}\int_A \left\{ D_{11}\left[\frac{\partial^2 w(x,y)}{\partial x^2}\right]^2 + D_{22}\left[\frac{\partial^2 w(x,y)}{\partial y^2}\right]^2 + 4D_{66}\left[\frac{\partial^2 w(x,y)}{\partial x \partial y}\right]^2 + 2D_{12}\frac{\partial^2 w(x,y)}{\partial x^2}\frac{\partial^2 w(x,y)}{\partial y^2}\right\} dA \quad (1)$$

where $D_{ij} = \int_{-h/2}^{h/2} Q_{ij}^{(k)} z^2 dz$ are the laminate stiffnesses, being $Q_{ij}^{(k)}$ the plane stress reduced stiffnesses of the kth lamina and h the thickness of the plate [19].

The maximum kinetic energy is given by [17]:

$$T_{\max} = \frac{1}{2}\omega^2 \int_A \rho h [w(x,y)]^2 \, dA \quad (2)$$

where ω is the angular frequency and ρ is the material density.

The Ritz method relies on the minimization of the functional $T_{\max} - U_{\max}$ with respect to the parameters W_{kl} [20,21]:

$$\frac{\partial (T_{\max} - U_{\max})}{\partial W_{kl}} = 0 \quad \text{with} \quad k = 1,\ldots,M \quad \text{and} \quad l = 1,\ldots,N \quad (3)$$

These parameters are the coefficients of a series defining the maximum amplitudes $w(x,y)$:

$$w(x,y) = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn} X_m(x) Y_n(y) \quad (4)$$

where $X_m(x)$ and $Y_n(y)$ are functions that verify the geometric boundary conditions, being M and N the number of terms in the series.

By replacing Eq. (4) in Eqs. (1) and (2) and applying Eq. (3) yields:

$$\sum_{m=1}^{M}\sum_{n=1}^{N}\left\{\int_A (\rho h X_k X_m Y_l Y_n) dA\right\} W_{mn}\omega^2 - \sum_{m=1}^{M}\sum_{n=1}^{N}\left\{\int_A \left[D_{11}\frac{d^2X_k}{dx^2}\frac{d^2X_m}{dx^2} Y_l Y_n + D_{22} X_k X_m \frac{d^2Y_l}{dy^2}\frac{d^2Y_n}{dy^2} + D_{12}\left(\frac{d^2X_k}{dx^2} X_m Y_l \frac{d^2Y_n}{dy^2} + X_k \frac{d^2X_m}{dx^2}\frac{d^2Y_l}{dy^2} Y_n\right) + 4D_{66}\frac{dX_k}{dx}\frac{dX_m}{dx}\frac{dY_l}{dy}\frac{dY_n}{dy}\right] dA\right\} W_{mn} = 0 \quad (5)$$

This equation defines an eigenvalue problem of size $M \times N$. In this work, a plate with all four edges clamped is studied and, therefore, are only considered functions that respect these boundary conditions. The functions proposed by Gartner and Olgac [22] where chosen for this study, since they present a greater numerical stability than the usual characteristic functions:

$$X_m(x) = A_m \cos\left(\frac{\gamma_m x}{a}\right) + B_m \sin\left(\frac{\gamma_m x}{a}\right) + C_m e^{-\frac{\gamma_m x}{a}} + D_m e^{-\frac{\gamma_m (a-x)}{a}}, \quad (6)$$

$$Y_n(y) = A_n \cos\left(\frac{\gamma_n y}{b}\right) + B_n \sin\left(\frac{\gamma_n y}{b}\right) + C_n e^{-\frac{\gamma_n y}{b}} + D_n e^{-\frac{\gamma_n (b-y)}{b}} \quad (7)$$

where a and b are the length and the width of the plate, respectively. The parameters A_r, B_r, C_r and D_r, for $r = m$ or n are given by:

$$A_r = 1, \quad B_r = -\frac{1 + (-1)^r e^{-\gamma_r}}{1 - (-1)^r e^{-\gamma_r}}, \quad C_r = -\frac{1}{1 - (-1)^r e^{-\gamma_r}}, \quad D_r = \frac{(-1)^r}{1 - (-1)^r e^{-\gamma_r}} \quad (8)$$

and the parameters γ_r are the solutions of the non-linear equation:

$$\cos(\gamma_r) - \frac{2e^{-\gamma_r}}{1 + e^{-2\gamma_r}} = 0 \quad (9)$$

Since the integrands in Eq. (5) are analytical functions, the integrals can be computed analytically, thus avoiding a discretization of the plate.

2.2. Damage model and localization method

In order to simulate the damage, a reference finite element model of the plate is created and its natural frequencies and mode shapes are computed. The damage considered in this work is defined by a reduction of the laminated stiffness $[D]$ of an element e, such that its Frobenius norm becomes:

$$\left\|[\widetilde{D}^{(e)}]\right\|_2 = \left(1 - d^{(e)}\right)\left\|[D^{(e)}]\right\|_2 \quad \text{with} \quad 0 \leqslant d^{(e)} \leqslant 1 \quad (10)$$

The severity of damage in the specified element is related with the parameter $d^{(e)}$ in the above equation. There is no reduction of the stiffness if this parameter is equal to zero, whereas if this parameter takes the value of one, there will be a complete reduction of the stiffness. The software FEAP was used to model the plate, using the SHEL1 finite element [23].

The damage localization indicators used in this work are the differences of modal displacement fields derivatives of damaged mode $\widetilde{w}_q$ and undamaged mode w_q. For the case of derivatives in the x direction these indicators can be written as [17]:

$$DFD_q^{(p)}(x,y) = \left|\frac{\partial^p \widetilde{w}_q(x,y)}{\partial x^p} - \frac{\partial^p w_q(x,y)}{\partial x^p}\right| \quad (11)$$

where p denotes the order of the spatial derivative and q the mode shape number.

The differentiation of the modal displacement fields of the undamaged plate $\partial^p w_q(x,y)/\partial x^p$ can be computed analytically, since it is defined by a series expansion (see Eq. (4)):