Computers and Geotechnics 74 (2016) 151–162

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Probabilistic analysis of a coal mine roadway including correlation control between model input parameters

Andrew J. Booth *, Alec M. Marshall, Rod Stace

Nottingham Centre for Geomechanics, Faculty of Engineering, The University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom

ARTICLE INFO

Article history:
Received 27 August 2015
Received in revised form 16 December 2015
Accepted 13 January 2016
Available online 4 February 2016

Keywords:
Coal mine
Correlation control
Latin hypercube
Monte Carlo
Probabilistic
Uncertainty analysis

ABSTRACT

Probabilistic analysis and numerical modelling techniques have been combined to analyse a deep coal mine roadway. Using the Monte Carlo method, a correlation control algorithm and a FLAC 2D finite difference model, a probability distribution of roof displacements has been calculated and compared to a set of measurements from an actual mine roadway. The importance of correlation between input parameters is also considered. The results show that the analysis performs relatively well, but does tend to over predict the magnitude of displacements. Correlation between parameters is shown to be very important, particularly between the three model stresses.

1. Introduction

There are two types of uncertainty; aleatory and epistemic [1]. In the context of geomechanics, aleatory uncertainty reflects underlying physical randomness and epistemic uncertainty a lack of knowledge about material and geometries [2,3]. Each must be treated differently; for example, it is possible to reduce epistemic uncertainty through field exploration or laboratory testing, in the sense that more information about a property reduces our uncertainty in it. However, more information will not reduce aleatory uncertainty [3]. This leads us to treating quantities in uncertainty analyses as random variables; that is, the quantity does not take a fixed value, but may assume any of a number of values, with it being impossible to know the true value until we observe it. Whitman [4] remarks that almost every factor dealt with in engineering analyses is truly a random variable, with some exhibiting more uncertainty than others.

Traditional design approaches are deterministic, where the lack of knowledge in the uncertainties is accounted for by introducing a factor of safety [5]. Analyses, sometimes referred to as reliability [5,6], probabilistic [7,8] or uncertainty [9,10], that try to account for the inherent variation in material properties, and other factors, are becoming more popular as the desire to assure satisfactory performance within the constraint of economy grows [5]. There are numerous approaches available, ranging in complexity and accuracy, to achieve these results, but what they all have in common is that they attempt to include the effects of property variability in a scientific way by using statistical methods [8].

In these approaches, all the model parameter inputs, whether they are, for example, dimensions, material properties, or stresses, are treated as random variables, expressed in the form of a probability density function (PDF). The aim of the exercise then becomes to estimate the PDF of some outcome, which is a function or dependent of the input random variables. It then becomes possible to give a probability to a certain value of the outcome occurring, such as the probability of failure, or more generally the probability of occurrence of an event.

One of the most widely used, and powerful, approaches to achieving this aim is the Monte Carlo method. First developed by researchers working on the development of the hydrogen bomb [11], the Monte Carlo method is now popular within fields ranging from finance to all manner of engineering. Baecher and Christian [2] divide Monte Carlo methods into two broad areas of use; firstly, the simulation of a process that is fundamentally stochastic, and secondly, problems that are not inherently stochastic but can be solved by simulation with random variables, such as the definite integration of a function. The work described in this paper falls into the first area.

In the field of geomechanics, Monte Carlo analyses are often combined with numerical modelling techniques [10,12–17]. In this

* Corresponding author.
E-mail addresses: andrew.booth@nottingham.ac.uk (A.J. Booth), alec.marshall@nottingham.ac.uk (A.M. Marshall), rod.stace@nottingham.ac.uk (R. Stace).

http://dx.doi.org/10.1016/j.compgeo.2016.01.008
0266-352X/

example of its use, the Monte Carlo analysis proceeds by solving a numerical model where the input parameter values are selected at random from their representative PDF, thereby treating each parameter as a random variable. The model is solved N times, where N may be in the hundreds or thousands, with the results usually being presented in the form of their own statistical distribution [7,10,18,19]. Monte Carlo simulation methods are primarily used in situations where there is uncertainty in the inputs and where the calculated uncertainty of results accurately reflects the uncertainty of the input data [20].

Despite finding widespread popularity in many research topics of geomechanics, such as slope stability [13,21–24], tunnelling [25–27] and open pit mining [28–30], there are surprisingly few examples of its application to deep coal mining [10,31]. Brown [32] states that uncertainty analysis techniques, such as Monte Carlo, have found popularity in open pit mining over underground mining due to "the fact that the geomechanics risks associated with a given underground mining project or operation, including cases of deep and high stress mining, are likely to be more wide-ranging and varied than those associated with bench, inter-ramp and overall slope stability in a given open pit mine". Canbulat [31] points to the difficulty in obtaining suitable and reliable distributions for the potentially many input parameters that can be required in an analysis of an underground mine.

Some researchers have recognised the value of Monte Carlo analysis in the design and planning of deep coal mines, especially given that the need to assure economic viability of mining operations is at an all-time high. Lu et al. [10] and Canbulat [31] have both combined the Monte Carlo analysis with numerical methods to aid in the design and planning of roadways serving longwall panels in a coal mine. However, considering the significant variability encountered in deep coal mines, for example within material properties [31,33] and stresses [34,35], there is certainly scope for increased use and development.

Approaches often used to account for the variation in material properties include the single random variable (SRV) and random field [2,8,36]. Of the two approaches SRV is the simplest as the material properties are treated as random variables that do not vary across their region of definition [2], only between each model run. In the random field approach the spatial variation of material properties is taken into account through a spatial correlation structure, based on the spatial correlation length of the material property. The spatial correlation length parameter describes the distance over which material property values will be significantly correlated [21,23] and is currently not very well documented [13,21].

As part of work being conducted on an ongoing EU Research Fund for Coal and Steel (RFCS) project, a planning tool that will allow mining engineers to assess how much of a deep coal mine roadway may need additional work to ensure stability and operational requirements is being developed. As part of the same project, data regarding coal mine properties is being collected. This data was used in this work to obtain reliable PDFs of input parameters, therefore addressing the issue raised by Canbulat [31] and reducing the inherent epistemic uncertainty. The planning tool will use Monte Carlo analysis with the SRV approach for modelling the variability of material properties, in conjunction with numerical modelling, to provide a probability distribution of roof displacements. By using this distribution, an engineer will be able to estimate the probability of displacements exceeding a certain threshold level over the entire length of a mine roadway and to plan the mine development accordingly.

This paper presents details of the development of this tool. Suitable methods of creating Monte Carlo samples are identified and studied. The importance of considering the correlation between cohesion and friction angle in a Mohr–Coulomb model is investigated, as well as between other parameters that have not been considered before, these being the three stress components of the numerical model.

2. Sampling for the Monte Carlo analysis

Simplistically, any Monte Carlo analysis can be considered as an evaluation of the function $\mathbf{y} = \mathbf{f}(\mathbf{x})$, where the function $\mathbf{f}$ is the model under study, $\mathbf{x} = [x_1, x_2, \ldots]$ is a vector of model inputs (also referred to as parameters or variables), each characterised by their own PDF, and $\mathbf{y} = [y_1, y_2, \ldots]$ is a vector of model predictions [9].

A sampling procedure must be chosen that generates samples from each input variable by randomly selecting values from each of their respective statistical distributions. In this way, analysis inputs are mapped to analysis results. Here, we consider two widely used sampling procedures; standard random sampling (SRS) and Latin Hypercube sampling (LHS). In SRS, the samples are generated independently from each PDF, with equal probability. The final sample takes the form

$$\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{i,nX}], \quad i = 1, 2, \ldots, nS, \tag{1}$$

where nX is the number of input variables and nS is the desired number of samples. A drawback of this method is that values can be sampled from any position on the PDF; therefore, particularly important, low probability but high consequence values will possibly be missed [9].

To overcome this problem, in an easily implementable manner, McKay et al. [18] introduced the LHS procedure. This method ensures that each of the input variables x_i has all portions of its distribution sampled from, by dividing each into nS strata of equal marginal probability $1/nS$. nS samples are taken from each x_i by randomly selecting one value from each strata. The nS values from x_1 are then paired at random with the nS values from x_2 and so forth until x_{nX}.

To calculate the actual values from the PDF of each input variable, the inverse cumulative distribution function (CDF) method can be used [37]. Using this technique, a randomly generated number u_i, from the uniform distribution $[0, 1]$, is entered into the equation of the Inverse CDF of each variable, which calculates a value from the distribution. This is why suitable theoretical PDFs for the variables are required, as the inverse CDF $F^{-1}(u_i)$ can be easily found. Both SRS and LHS use this method to generate the values, the only difference being that in SRS the current random number can be generated from anywhere on the distribution $[0, 1]$, whereas in LHS it must come from the current strata. One of the most important factors for carrying out good Monte Carlo simulations is a mechanism that can produce long sequences of random numbers u_i [37]. To achieve this, a pseudorandom number generator is used. These generators are actually deterministic algorithms and will therefore eventually produce repetition in the sequence of random numbers. To prevent this affecting the analysis, they should possess a very long period relative to the number of samples nS, where the period is the length of the sequence before repetition occurs.

A computer programme, written in the C++ language, has been developed to create samples for Monte Carlo analysis. The programme can generate samples from numerous theoretical distributions, using either the SRS or LHS procedure combined with the Inverse CDF method of calculating values; in this work the LHS procedure is used. The Mersenne-Twister [38] pseudorandom number generator is used in the programme to generate the required sequences of random numbers. This algorithm is particularly well suited to Monte Carlo analyses due to its extremely long period of $2^{19937} - 1$.