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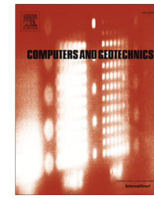

Computational dynamic homogenization for the analysis of dispersive waves in layered rock masses with periodic fractures

A. Bacigalupo, L. Gambarotta *

Department of Civil, Chemical and Environmental Engineering, University of Genoa, via Montallegro 1, 16145 Genoa, Italy

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ABSTRACT

The analysis of the wave propagation in layered rocks masses with periodic fractures is tackled via a two-scale approach in order to consider shape and size of the rock inhomogeneities. To match the displacement fields at the two scales, an approximation of the micro-displacement field is assumed that depends on the first and second gradients of the macro-displacement through micro-fluctuation displacement functions obtained by the finite element solution of cell problems derived by the classical asymptotic homogenization. The resulting equations of motion of the equivalent continuum at the macro-scale result to be not local in space, thus a dispersive wave propagation is obtained from the model. The simplifying hypotheses assumed in the multi-scale kinematics limit the validity of the model to the first dispersive branch in the frequency spectrum corresponding to the lowest modes.

Although the homogenization procedure is developed to study the macro-scale wave propagation in rock masses with bounded domain, the reliability of the proposed method has been evaluated in the examples by considering unbounded rock masses and by comparing the dispersion curves provided by the rigorous process of Floquet–Bloch with those obtained by the method presented. The accuracy of the method is analyzed for compressional and shear waves propagating in the intact-layered rocks along the orthotropic axes. Therefore, the influence of crack density in the layered rock mass has been analyzed. Vertical cracks have been considered, periodically located in the stiffer layer, and two different crack densities have been analyzed, which are differentiated in the crack spacing. A good agreement is obtained in case of compressional waves travelling along the layering direction and in case of both shear and compressional waves normal to the layering. The comparison between two crack systems with different spacing has shown this aspect to have a remarkable effect on waves travelling along the direction of layering, and limited in the case of waves propagating normal to the layers.

The equivalent continuous model obtained through the dynamic homogenization technique here presented may be applied to the computational analysis of non-stationary wave propagation in rock masses of finite size, also consisting of sub-domains with different macro-mechanical characteristics. This avoids the use of computational models represented at the scale of the heterogeneities, which may be too burdensome or even unfeasible.

1. Introduction

It is well known that fractures and layering in rock masses have a large impact on both the mechanical and hydraulic behavior of rock masses. Notably, the overall properties of the rock systems depend not only on the mechanical parameters of the intact rock but also on the joints and fractures behavior and distribution (i.e. their spacing, length, condition, orientation, continuity and the number of joint sets) and on the layering morphology (see for Refs. [21,8,18,13]). This point is remarkable when seismic elastic waves are considered because fractures and joints can trap and guide waves and their behavior may prove useful for probing the geometrical and mechanical properties of the fractures [6].

A large number of studies on wave propagation in fractured rock masses have been developed over the past two decades. Although it is well known that in fractured and porous reservoirs the effect of saturating fluid on wave propagation may be significant (see [15]), because the fluid may cause significant frequency dependent attenuation and dispersion, these effects also occur in drained conditions in case of layered or fractured rocks [19]. Based on the displacement discontinuity approach [22], different contributions to the understanding of the influence of rock cracking on wave propagation have been given (see [8], among the others). In addition, computational approaches have been proposed, including the distinct element method that has aroused considerable interest

* Corresponding author.
E-mail address: luigi.gambarotta@unige.it (L. Gambarotta).

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(see [16,25]). A complementary approach is based on the description of the cracked rock as an equivalent continuum whose effective parameters are derived by homogenization techniques (see for Ref. [14]). In this context, the choice of the equivalent continuum to be considered depends on the wavelength compared to the characteristic size of the rock meso-structure. In the long-wavelength limit, when the size of the seismic wavelength is much larger than the layer and/or fracture spacing, the wave propagation may be analyzed by the effective medium theory, i.e. the classical Cauchy model. However, when the wavelength becomes smaller, a dispersive behavior is obtained that may be significantly affected by the layer and fracture spacing. If rock masses with stratification and periodic cracking are considered, the propagation analysis of dispersive waves can be performed through the classical approach of Floquet–Bloch at the periodic cell level. However, in the more general case of wave propagation in bounded domains the analysis of the heterogeneous model would be very labor intensive so that multi-scale approaches based on non-local homogenization techniques are preferred (see [10,9,2,3]).

In the present paper, a dynamic homogenization procedure is addressed to the study of layered rock systems. This procedure (also developed in [3] and applied to layered materials in analytical form) is an enhancement of a previous approach already proposed by the Authors [2]. In this contribution the overall second order elastic moduli and inertia terms are obtained as the result of a variational-asymptotic homogenization based on a proper down-scaling law, in which the fluctuation field at the micro-scale is assumed in a proper form that satisfies the continuity at the interface of adjacent cells. To evaluate the accuracy of the dynamic model defined in the equivalent homogeneous continuum, a periodic stratified rocky system is considered. The dispersion curves of shear and compressional waves along and transversely to the direction of the layers obtained by this model are compared with those by the rigorous approach by Floquet–Bloch. Therefore, in order to catch the influence of cracks on the dynamic response, the case of stratified rocks with periodic fissuring of rigid layers in agreement with [12,4], is considered. Two different crack spacings are considered with the dual purpose to compare these results with those of the intact rock system and to appreciate the influence of the ratio between the crack spacing and thickness of the layers.

2. The micro-scale dynamic model for elastic periodic layered rock masses

A 2-D description of the layered rock mass is considered as an elastic periodic bimaterial undergoing plane strain deformation within the classical elasticity theory (Fig. 1). A periodic horizontal distribution of vertical cracks is assumed as given in the phases. As a consequence the resulting periodic material may be fully characterized by the periodic cell $\mathbf{A} = [0, \delta\varepsilon] \times [0, \varepsilon]$ with characteristic size ε shown in Fig. 1b, which is spanned by the two independent orthogonal vectors $\mathbf{v}_1 = d_1\mathbf{e}_1 = \delta\varepsilon\mathbf{e}_1$, $\mathbf{v}_2 = d_2\,\mathbf{e}_2 = \varepsilon\mathbf{e}_2$. The elasticity tensor $\mathbb{C}^{m,\varepsilon}(\mathbf{x})$ and the mass density $\rho^{\varepsilon}(\mathbf{x})$ are $\mathbf{A}$-periodic, i.e. $\mathbb{C}^{m,\varepsilon}(\mathbf{x}+\mathbf{v}_i) = \mathbb{C}^{m,\varepsilon}(\mathbf{x}), \rho^{\varepsilon}(\mathbf{x}+\mathbf{v}_i) = \rho^{\varepsilon}(\mathbf{x}), \quad i = 1,2, \forall \mathbf{x} \in A$ (the material point is identified by vector $\mathbf{x} = \{x_1\ x_2\}^T$). Moreover, a simplified description of the cracks is assumed where both normal and tangential displacement jumps across the crack faces are allowed, without considering unilateral and frictional effects. This traction-free boundary condition on the crack faces is obtained by modelling the cracks as rectangular regions having small width and vanishing elastic stiffness. This suggest to consider a unit cell $Q = [0, \delta] \times [0, 1]$ that reproduces the periodic microstructure by rescaling with the small parameter ε so that the two distinct scales are represented by the macroscopic (slow) variable $\mathbf{x} \in \mathcal{A}$ and the microscopic (fast) variable $\xi = \mathbf{x}/\varepsilon \in Q$. The mapping of both the elasticity tensor and of the mass density may be defined on Q as follows: $\mathbb{C}^{m,\varepsilon}(\mathbf{x}) = \mathbb{C}^{m}(\xi = \mathbf{x}/\varepsilon), \rho^{\varepsilon}(\mathbf{x}) = \rho(\xi = \mathbf{x}/\varepsilon)$, respectively. The (micro) displacement $\mathbf{u}(\mathbf{x}, t)$ of a material point $\mathbf{x}$ at time t is considered together with the corresponding (micro) strain tensor $\boldsymbol{\varepsilon}(\mathbf{x}, t) = sym\nabla\mathbf{u}(\mathbf{x}, t)$. Moreover, the (micro) stress tensor is given by the elastic constitutive equation $\boldsymbol{\sigma}(\mathbf{x},t) = \mathbb{C}^m\left(\frac{\mathbf{x}}{\varepsilon}\right)\boldsymbol{\varepsilon}(\mathbf{x},t)$ and it has to satisfy the local equation of motion $div\boldsymbol{\sigma}(\mathbf{x},t) = \rho\left(\frac{\mathbf{x}}{\varepsilon}\right)\ddot{\mathbf{u}}(\mathbf{x},t) - \mathbf{f}(\mathbf{x},t)$, where $\mathbf{f}(\mathbf{x},t)$ is the body force. The resulting set of partial differential equations is written in the form

$$div\left(\mathbb{C}^m\left(\frac{\mathbf{x}}{\varepsilon}\right)\nabla\mathbf{u}(\mathbf{x},t)\right) = \rho\left(\frac{\mathbf{x}}{\varepsilon}\right)\ddot{\mathbf{u}}(\mathbf{x},t) - \mathbf{f}(\mathbf{x},t), \tag{1}$$

with the fourth-order elasticity tensor having the property $\mathbb{C}^m\mathbf{Z} = \mathbb{C}^m sym\mathbf{Z}, \forall\mathbf{Z}$.

It should be noted that the two-dimensional model described above could be extended to the more general case of periodicity of the rock mass also along axis x_3 (with periodicity vector $\mathbf{v}_3$). In this case, the cell is represented by a periodic rectangular cuboid and the equation of motion (1) is written in components according to the three orthogonal directions. In fact, when the structure of the rock mass is not uniform along axis x_3, but periodic, the motion is no longer in-plane and inertial effects appear also along direction x_3. The two-dimensional model cannot directly describe this condition and a three-dimensional model is needed that is far more computationally onerous.

3. The macro-scale second-order dynamic model

From the equation of motion (1) it is convenient to express the micro-displacement field $\mathbf{u}(\mathbf{x}, \xi = \frac{\mathbf{x}}{\varepsilon}, t)$ as a function of both the slow $\mathbf{x}$ and the fast ξ variables, respectively, as usually is made in asymptotic homogenization. Moreover, in the following, the micro-displacement is considered to be L-periodic, being $L = [0, \delta L] \times [0, L]$ with L fixed such that L/ε is a large integer number. This condition is fulfilled in the case of L-periodic body

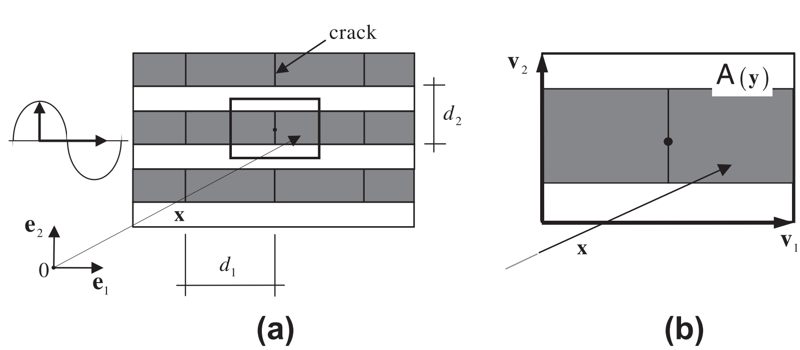

Fig. 1. (a) Fissured layered rock mass with periodic structure; (b) unit cell and periodicity vectors.