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Technical note

A new Lagrangian analysis method used to study dynamic performance of concrete with multiple velocity gauges in attenuating waves

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ABSTRACT

Lagrangian analysis method derives the specific internal energy, strain, and stress and particle velocity histories from either stress or particle velocity records from a series of gauges embedded in material. However, if only particle velocity is obtained, the existing Lagragian analysis methods need to know stress at initial Lagrangian position. In this paper, a new Lagrangian analysis method for multiple particle velocity gauges (termed path-line time stepping) is presented. The method can derives the stress from particle velocity records without any initial conditions. Besides, the method is applied to measure waves in concrete. And the verification of the analysis for attenuating waves is studied by numerical simulation that reproduced the stress-strain histories.

1. Introduction

The importance of safety in modern impact technology is widely appreciated. Consequently, constitutive model of solid materials at high strain rates attracts increasingly interests [1–5]. The Lagrangian analysis method is the implantation of a series of two or more stress or particle velocity or strain gauges in the test material to record the passage of the disturbance. The gauges are termed "Lagrangian" because they move with the material. Lagrangian analysis method is mainly numerical technique for treating the resulting gauge records to direct obtain the stress-strain relations for the material. This method is applicable to arbitrarily complex materials and has been widely used to study the dynamic mechanical response of materials subject to impact loading.

Lagrangian analysis method was first introduced by Fowles R. and Williams R. F. [6,7]. They deduced the time-dependent constitutive relation of materials by the so-called Lagrangian analysis without introducing additional assumption of constitutive relation. However, the Fowles-Williams method is validated only when all gauges reach the same peak value without wave decaying. Cowperthwaite and Williams provided a generalization of the Fowles-Wiliams method. Their result [8] can be applied to the data in which there is an attenuation of the peak stress or particle velocity. Grady [9] introduced a concept of the path lines as an aid in computing derivatives for attenuating flow. Based on the path-line method, Seaman [10] developed the surface fitting method and assumed that the fitting surface is monotonous and smooth, and the third-order partial derivatives are equal to zero ($d^3\sigma/dh^3 \equiv 0$). Lagrangian inverse analysis [11–13] avoided Seaman assumption and obtains the stress of the flow field. However, the result depends on the assumed form of the unknown stress function which may produce non-unique solution. This method has been denied by Seaman. To verify the reliability of the computed results, Gupta [14] proposed a self-consistent test method to calculate the stress waves from the particle velocity waves and then obtain the particle velocity waves from the calculated stress waves by inverse operation. Forest [15] proposed the impulse time integral method for data processing and variance estimation. However, it is still difficult to establish the precise function form when the measurement does not involve the stress. Wang et al. [16] proposed a new method that combines the Lagrangian analysis with the Hopkinson pressure bar (HPB) technique. However, the initial strain still depends on mathematical processing. However. Since the 1970s to 2010s, a few articles [17–21] continued to improve the numerical technique and engineering applications of Lagrangian analysis method.

Lagrangian analysis method can be used to analyze currents and flows of various materials [22,23] by analyzing data collected from gauges embedded in the material which move freely with the motion of the material. Therefore, Lagrangian analysis method is mainly adapted to study the materials strain rate-dependent

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elastic–plastic constitutive characteristics. Specifically, there are three solutions.

(1) Once one-dimensional stress histories $\sigma(h_k, t)$ $(k = 1, 2, \cdots m)$ were measured by m stress/pressure gauges embedded in the specimen at m Lagrangian positions h_k, it is easy to determine the dynamic stress-strain curves by the traditional Lagrangian analysis method. In fact, with the measured stress $\sigma(h_k, t)$, the corresponding first-order partial derivatives of stress $\sigma(h_k, t)$ with respect to time t and Lagrangian positions h_k can be numerically calculated. According to the momentum conservation equation, the first-order partial derivatives of particle velocity $u(h_k, t)$ with respect to t can be determined. Usually, the initial condition $u(h_k, 0) = 0$ is known. Thus, the particle velocity $u(h_k, t)$ at different Lagrangian positions h_k can be integrated through time t. Similarly, the strain $\varepsilon(h_k, t)$ at h_k can be determined from the mass conservation equation. Finally, the stress $\sigma(h_k, t)$ and the deduced $\varepsilon(h_k, t)$ at h_k can be obtained.

(2) However, if particle velocity histories $u(h_k, t)$ have been measured by m velocity gauges embedded in the specimen at m Lagrangian positions h_k, it is difficult to solve the stress $\sigma(h_k, t)$ and strain $\varepsilon(h_k, t)$ at h_k. The integration cannot be performed along the path-line due to no initial value for stress $\sigma(h_k, t)$. Thus, in order to completely determine the stress $\sigma(h_k, t)$ in the flow field, a known stress history is required at some point in the flow. It means that a simultaneous measurement of particle velocity $u(h_k, t)$ and the initial stress $\sigma(0, t)$ is required. By doing so, the strain $\varepsilon(h_k, t)$ at h_k can be determined from the mass conservation equation.

(3) Furthermore, if strain histories $\varepsilon(h_k, t)$ are measured by m velocity gauges embedded in the specimen at m Lagrangian positions h_k, it becomes more difficult to determine the stress $\sigma(h_k, t)$ and velocity $u(h_k, t)$ at h_k. The reason is that the initial conditions $\sigma(0, t)$ and $u(0, t)$ are all unknown. It means that a simultaneous measurement of strain histories $\varepsilon(h_k, t)$, the initial stress $\sigma(0, t)$ and particle velocity $u(0, t)$ is required. It is seen that the third case is the most complex and difficult one due to two required initial conditions.

The traditional Lagrangian analysis method is for the first case that derives the physical quantities' histories with the stress measured in experiment. However, the time response of the particle velocity gauges are greater than the manganin gauge, and the valid records is longer. So, the particle velocity gauge is more suitable for measuring the passage of the disturbance. In the following paragraphs we describe the analysis procedure that only need particle velocity histories. The analysis derives the stress and strain histories from the particle velocity records without introducing additional assumptions. Two advantages will be achieved in this analysis. (1) It guarantees the uniqueness of the data. (2) Only the first-order derivatives and partial derivatives of the physical quantity are involved, which greatly reduces the impact of the local jitter of the test data on processing results.

2. Lagrangian analysis method

A detailed description of the basic principle of the Lagrangian analysis had been made by Fowles in 1973. For convenience, we consider a case of one-dimensional stress waves propagating in a rate-dependent material, which corresponds to the propagation of time-dependent non-simple waves with attenuation and dissipation characters. So the Lagrangian analysis is mainly based on the following three conservation equations without any assumption of material constitutive relation. In Lagrangian coordinates these relations are:

$$\rho_0\left(\frac{\partial u}{\partial t}\right)_h + \left(\frac{\partial \sigma}{\partial h}\right)_t = 0 \tag{1}$$

$$\left(\frac{\partial \varepsilon}{\partial t}\right)_h + \left(\frac{\partial u}{\partial h}\right)_t = 0 \tag{2}$$

$$\left(\frac{\partial E}{\partial t}\right)_h + \frac{\sigma}{\rho_0}\left(\frac{\partial u}{\partial h}\right)_t = 0 \tag{3}$$

where σ is the stress in the direction of propagation, u is the particle velocity, ε is the strain, E is the specific internal energy, h and t are the Lagrangian position and time respectively, ρ_0 is the initial density. The conservation equation of momentum maybe describe a relation between the partial derivative of stress σ with respect to h and the partial derivative of particle velocity u with respect to t. The conservation equation of mass maybe give a relation between the partial derivatives of strain ε with respect t and the partial derivative of particle velocity u with respect to h. The conservation equation of energy maybe show a relation between the partial derivatives of specific internal energy E with respect to t, stress σ and the partial derivatives of particle velocity u with respect to h. As the result, the basic conservation equations give a relation between dynamic stress σ, strain ε and particle velocity u. And the relationship between stress σ and strain ε must be given by the particle velocity u.

The variables connected by conservation Eqs. (1), (2) and (3) are not the stress σ, strain ε and particle velocity u themselves but the first-order partial derivatives of them. Thus, in order to derives the stress σ, strain ε and particle velocity u from either stress σ or particle velocity u, the preceding equations are required to integrate along lines of constant h (the particle line).

Eqs. (1)-(3) requires development of methods to (i) smooth the raw gauge records; (ii) divide the records into discrete time intervals; (iii) construct the path-line as an aid in computing derivatives needed for attenuating flow; (iv) numerically evaluate the partial derivatives; (v) perform the integrations. The approach can be visualized with the aid of Fig. 1 which shows a series of smoothed velocity histories above a Lagrangian distance-time (h-t) plane.

The original calibrated velocity-time data u^*, t^* obtained from the gauge records are replaced by a series of points $u_{J,k}$, $t_{J,k}$, which

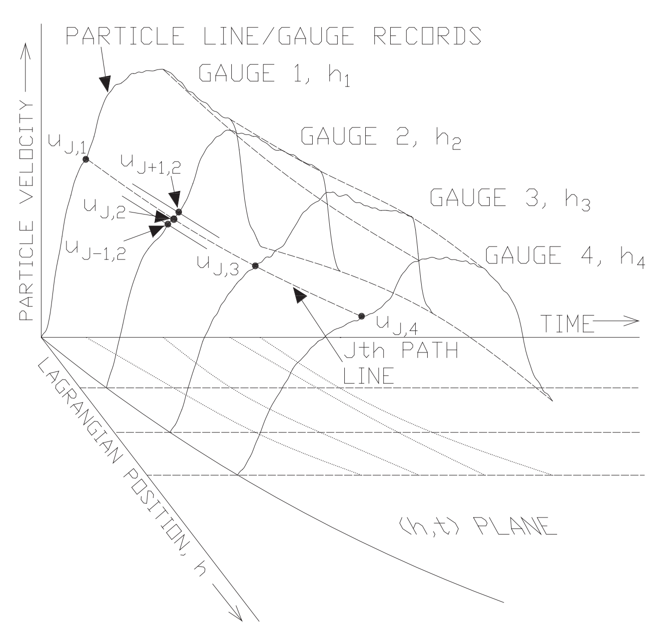

Fig. 1. Lagrangian particle velocity gauge records.