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Exact matching at a joint of multiply-connected box beams under out-of-plane bending and torsion

Soomin Choi, Yoon Young Kim *

School of Mechanical and Aerospace Engineering and Institute of Advanced Machines and Design, Seoul National University, Gwanak-ro 1, Gwanak-gu, Seoul 151-742, Republic of Korea

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ABSTRACT

Accurate analysis of thin-walled box beams meeting at a joint requires not only consideration of higher-order deformation degrees (such as warping and distortion) but also exact matching conditions at the joint. Especially when more than two box beams are connected at a joint, the deformation of the beam-joint system is so complicated that no one-dimensional beam analysis has yet predicted its structural behavior correctly. Since a beam theory incorporating higher-order deformations is available, the main difficulty is determining the exact matching conditions at the joint. In this paper, we derive the exact matching conditions for five field variables—bending deflection, bending/torsional rotations, warping, and distortion—of multiply connected box beams under out-of-plane bending and torsional loads. The derived relations are valid irrespective of the number of beams and angles. We introduced a new concept called "edge resultants" besides conventional (sectional) resultants, and demonstrated its effectiveness for exact derivation and physical interpretations of the derived equations. The accuracy and validity of the proposed theory are checked by comparing the predicted results with those of shell finite element analysis.

1. Introduction

Compared with the results of classical Euler and Timoshenko beam theories (e.g., [1,2]), thin-walled box beams behave more flexibly. This is because cross-sectional deformations not covered by those classical theories are evident in the box beams. When box beams meet at a joint, the magnitudes of cross-sectional deformations near the joint are amplified, which causes the joint region to exhibit significant flexibilities. For this reason, the behavior of box beam systems that consist of box beam members and joints differs markedly from the predictions of classical beam theories. To overcome this overestimation of stiffness, one-dimensional higher-order beam theories that include cross-sectional deformations as additional degrees of freedom have been developed [3–10]. In higher-order beam theories, however, determining the matching relations among all degrees of freedom at a joint is difficult because higher-order deformation degrees such as warping and distortion do not produce non-zero resultants. Especially for box beam systems subjected to out-of-plane bending and torsion, distortion is complicatedly coupled with other degrees of freedom at the joint. However, there has been no dedicated research to investigate how they are coupled. In fact, no exact analysis method based on higher-order beam theories is applicable to systems of "three" or more box beams connected at a joint, under out-of-plane loads. Therefore, we propose the first exact analysis approach for cases of three or more box beam-joint systems under out-of-plane loads.

Previous studies have attempted to express the joint flexibilities of box beam systems using one-dimensional beam theories. Initial studies, based mainly on classical beam theories, introduced joint models including rotational springs to account for those flexibilities [11–14]. El-Sayed [11] proposed a joint model consisting of torsional springs to represent the flexibilities observed under out-of-plane loads, and Lee and Nikolaidis [12] proposed a modeling technique using springs and rigid sections to consider additional joint coupling effects. Nonlinear joint elements were proposed in Refs. [13,14] to model the effect of flexible joint nonlinearity on structural response under cyclic and dynamic actions. Meanwhile, Becker et al. [15] suggested an approach to evaluate the stiffness of a joint from dynamic responses. Recently, joint concept modeling approaches were proposed in Refs. [16,17] that reduce the shell element-based detailed joint into a super element through static or dynamic reduction techniques. However, joint flexibilities vary considerably depending on both the number of connected box beam members and the joint angles. Thus, it is

* Corresponding author.
E-mail addresses: soomin0603@snu.ac.kr (S. Choi), yykim@snu.ac.kr (Y.Y. Kim).

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difficult to develop a consistent joint model applicable to various joints using classical beam theories.

A beam theory that considered the significant cross-sectional deformations as additional degrees of freedom could determine the flexibilities of thin-walled box beam members or systems without employing artificial concepts. Vlasov [3] theoretically defined the warping deformation under twisting moment as a sectorial coordinate, and established a beam theory for thin-walled beams that includes warping as a higher-order deformation degree. For stress analysis, buckling analysis, dynamic analysis, etc., using advanced beam theories, several analytic or semi-analytic methods have been proposed; e.g., one based on Saint Venant's theory [18,19], the variational asymptotic method [20–22], Carrera's unified formulation [23,24], and the GBT cross-section analysis [25–27]. For thin-walled closed section beams including box beams, Kim and Kim [6,28–31] developed a higher-order beam theory that interprets their torsional behavior correctly. In this regard, they recognized the importance of considering distortional deformations in addition to warping deformations and proposed a semi-analytic method to define those cross-sectional modes. In recent years, higher-order beam models have been developed to analyze the stress distribution or nonlinear behavior of thin-walled box beam members. Genoese et al. [8,32] proposed a mixed beam model considering warping modes derived from their Saint Venant theory-based approach and a mixed formulation with independent descriptions of stress and displacement fields. Ferradi and Cespedes [9,33] proposed a method that calculated distortion modes by modal analysis of a cross-section decomposed with beam elements and derived relevant warping modes using their equilibrium scheme. Vieira et al. [10,34] derived a generalized eigenvalue problem that defined uncoupled warping modes under the assumption of in-plane rigid cross-sections and suggested a higher-order beam model considering those warping modes. Some extended results that model the nonlinear response of steel-concrete composite members can be also found in Refs. [35,36].

Establishment of higher-order beam theories including the effects of cross-sectional deformations was followed by efforts to theoretically express the joint flexibilities of thin-walled beam systems. Especially concerning thin-walled open section beams, many definitions of the joint compatibility of degrees of freedom have been proposed [37–43]. Vacharajittiphan and Trahair [37] investigated the warping restraint/transmission at the joint of two I-section members and found that distortion influenced the warping transmission. Baigent and Hancock [38] determined the equilibrium condition at the joint of two asymmetric section members by transforming force terms on the centroid and the shear center to the member origin axes and derived corresponding displacement relations at the joint including warping coupling effects. In addition, they proposed a modeling technique to consider the effects of different joint types and eccentric restraint. Based on the above research, Basaglia et al. [41] recently derived extended displacement relations applicable to a joint of multiple open-section members and determined the warping transmission for various joint types. Considering additional displacement constraints at specific points of the joint, they established a Generalized Beam Theory (GBT)-based approach that interprets various buckling behavior of thin-walled open section beam systems [42].

The flexibility at a joint of thin-walled closed section members is induced by distortional deformation. Thus, considering the effects of distortion as well as warping on the joint flexibilities is required. Especially for box beam systems under out-of-plane loads, joint flexibilities are caused mainly by the coupling of distortion with other degrees of freedom because the location of the centroid is identical to that of the shear center. Therefore, effort has focused on defining those effects of distortion so as to express correctly the joint flexibilities of box beam systems under out-of-plane loads [44–48]. Solving an optimization problem that minimizes differences between the displacements of two box beams on their imaginary joint section, Jang et al. [44–46] determined displacement matching conditions at a joint of two box beams. For the same two box beam-joint systems, Choi et al. [47] theoretically derived exact joint-matching conditions to capture joint behavior comparable with that predicted by detailed shell analysis. The methods (for two box beam-joint systems) proposed in Refs. [44–47] may be used in more general cases; i.e., three or more box beam-joint systems. However, the joint stiffness is overestimated because the joint is excessively constrained and higher-order deformations, such as warping and distortion, cannot develop at the joint.

To develop a precise higher-order beam analysis method applicable to three or more box beam-joint systems, a new approach to theoretically derive joint-matching conditions is required. Especially when the joint is defined as a gathering point of box beam members, as used in the Euler/Timoshenko beam and box beam [44–47] theories, the exact matching conditions defined at that point have not been determined. From the observation that two adjacent box beam members always share one common edge near the joint, Jang et al. [48] recently proposed joint-matching conditions that satisfy three-dimensional displacement continuity between those two members along the actual location of the common edge. Using those conditions, they analyzed three box beam-joint systems under out-of-plane loads. Since the joint is described as several scattered points, however, equilibriums of the resultant forces or moments cannot be held exactly at the joint, which results in errors in the analyses.

In this study, three or more box beam-joint systems under out-of-plane bending and torsion are analyzed using a higher-order beam theory. The unique contribution of this investigation is the derivation of the exact matching relations among all field variables of box beam members meeting at the joint. Fig. 1 shows a three or more box beam-joint system. Only a portion of the system—such as Beam $i-1$, Beam i, and Beam $i+1$ ($i \geqslant 2$)—is depicted, for convenience. It is assumed that all box beams in Fig. 1 are on the same plane, and that their width, height, and thickness are equal to b, h, and t, respectively. To interpret the box beam-joint system depicted in Fig. 1 using the higher-order beam theory, the connectivity between box beams is modeled in Fig. 2. As with the classical beam theories and Refs. [44–47], the point at which all box beams converge is defined as the joint (strictly speaking, the joint refers to the point at which the central axes of box beams meet). Shared Side Edge $i-1$ in Fig. 1, which is shared by Beam $i-1$ and Beam i ($i \geqslant 2$), is extended and represented in Fig. 2 by Edge $M_{i-1}M'_{i-1}$ in Beam $i-1$ and Edge $N_iN'_i$ in Beam i. So Edge $M_{i-1}M'_{i-1}$ and Edge $N_iN'_i$ are regarded in this study as being rigidly connected to each other (by an imaginary rigid body). Therefore, although Edge

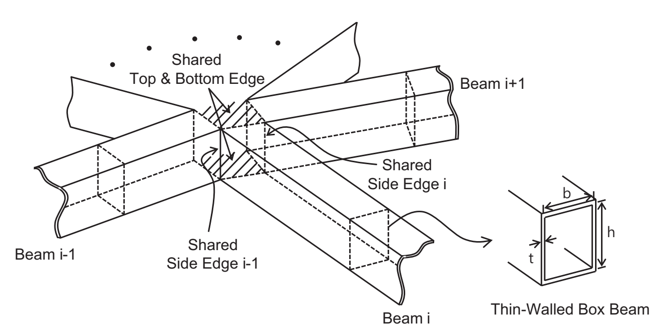

Fig. 1. Three or more thin-walled box beam-joint systems (only a portion of the system such as Beam $i-1$, Beam i, and Beam $i+1$ ($i \geqslant 2$), is depicted).