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Short communication

Application of Timoshenko beam theory to the estimation of structural response

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ABSTRACT

Multi-story buildings are usually instrumented at a limited number of floors. Consequently, the structural response (e.g., acceleration, displacement) at the non-instrumented floors remains unknown and needs to be estimated using the acceleration data recorded at the instrumented floors. The prevailing way to estimate the unknown structural response is to interpolate the recorded data over the height of the building. Other methods such as the Mode Shape Based Estimation (MSBE) and Timoshenko Beam Based Estimation (TBBE) methods, on the other hand, use the mode shapes of bending, shear and Timoshenko beam modeling to estimate the unknown structural response. The Factor Building at the UCLA campus in Los Angeles, California, USA is utilized to test the performance of these methods, and the results are compared with conventional interpolation methods (e.g., linear or cubic spline interpolations). The results show clearly that both the MSBE and the TBBE methods provide a better estimation of unknown structural response than both linear interpolation methods.

1. Introduction

Multi-story buildings are usually instrumented at a limited number of floors, and consequently the unknown structural response (e.g., acceleration, displacement) at non-instrumented floors must be estimated using the acceleration data recorded at instrumented floors. Many methods have been developed over the years to calculate the structural response at non-instrumented floor levels. The conventional way to calculate such unknown structural response is to interpolate the recorded structural response over the height of the building using the linear and cubic polynomial (spline) interpolation techniques [14,15,7]. The cubic spline interpolation method prosed by Beatson [1] is adopted in this work to estimate the motions at non-instrumented floors. Both of the interpolation techniques, on the other hand, require that a sensor must be installed both at the base and the roof of the building, as well as at some intermediate floors, in order to obtain an acceptable estimate of structural response at non-instrumented floors. Kaya et al. [10] proposed a new simplified, Mode Shape Based Estimation (MSBE) method, which estimates the vibration time histories of non-instrumented floors using the records at instrumented floors. This method calculates the contribution of shear- and bending-beam modes to each mode shape of the multi-story building. This paper presents the Timoshenko Beam Based Estimation (TBBE) method, an alternate method to estimate the unknown structural response using the mode shapes of the Timoshenko Beam only. The differences in the beam theory used in the derivation of each of these beam models is discussed later. Timoshenko beam has been extensively used in literature to simulate building motions [2], to predict the propagation of waves in buildings [3], and in the performance of system identification [6].

The response of the proposed TBBE method is tested, alongside the response of the previously proposed MSBE method, and linear and cubic polynomial interpolation methods, using data from two earthquakes recorded at UCLA's Doris and Louis Factor Building. The performance of each of the four methods is compared and discussed together in the following sections.

2. Mode shapes of bending, shear, and Timoshenko beams

The most commonly used beam model, the Euler–Bernoulli beam assumes that the bending effect is the single most important factor in transversely vibrating beams, and considers in its formulation the strain energy due to the bending and the kinetic energy due to lateral displacements; however, it does not take into account shear distortions. The shear beam model adds the effects of shear distortions to the Euler–Bernoulli beam model. The Timoshenko beam model builds on this and adds the effects of shear distortions and the effect of rotation of the cross-section to

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the Euler–Bernoulli beam. The differential equations of motion of the Euler–Bernoulli, shear and Timoshenko beams as well as the details of the derivations of a general solution to the mode shapes of a bending-beam, $\phi_b(x)$, of a shear-beam, $\phi_s(x)$, and of a Timoshenko beam, $\phi_t(x)$ are given in Han et al. [8].

The first four normalized mode shapes of the inverted cantilever bending, shear, and Timoshenko beams are plotted in Fig. 1. Clamped at one end and free at the other end, the inverted cantilever beam used in this paper has a uniform rectangular cross section with a unit length of 1 m, a shape (shear) factor of 0.40, a Poisson's ratio of 0.29, and a slenderness ratio of 6.92. There are, for example, combinations of these properties for which the calculated response of a beam using shear and Timoshenko beam theory may be identical. This particular situation was carefully avoided by selecting these properties; moreover, the response of the Timoshenko beam using these properties is analogous to a high-rise building [5].

Due to differences in boundary conditions and the additional effects of shear distortions in the shear-beam model as well as the rotation of the cross-section, all of the beam models result in different mode shapes. It should be noted that both the curvature and the amplitude of the mode shapes are different for all modes, and that the local peaks of each mode shape along the length of the beam for all of the beam models do not occur at the same location.

The MSBE method assumes that the mode shape of a building can be estimated as a linear combination of the mode shapes of a shear beam and a bending beam whereas the TBBE method uses the mode shape of a Timoshenko beam only as shown in Eqs. (1a) and (1b), respectively.

$$\phi_{j,k} = C_{s,j} \cdot \phi_{s,j,k} + C_{b,j} \cdot \phi_{b,j,k} \tag{1a}$$

$$\phi_{j,k} = C_{t,j} \cdot \phi_{t,j,k} \tag{1b}$$

where $\phi_{s,j,k}$, $\phi_{b,j,k}$ and $\phi_{t,j,k}$ are the amplitudes of the jth mode shapes of a shear beam, bending beam, and Timoshenko beam, respectively, at the kth floor; $\phi_{j,k}$ is the amplitude of the jth mode shape of the building at the kth floor; $C_{s,j}$, $C_{b,j}$ and $C_{t,j}$ are the jth mode unknown weighting coefficients of a shear-beam, bending beam, and Timoshenko beam, respectively. The solution of the differential equation in Han et al. [8] assumes that both the mass and the stiffness of the shear, bending, and Timoshenko beams are uniformly distributed; therefore, the MSBE and the TBBE methods should only be applied to those multi-story buildings whose mass and stiffness distribution are uniform or close to uniform along the height of the building. The error in the estimation for the jth mode can be expressed as the square sum of the differences over the instrumented floors between the calculated modal acceleration, $\ddot{z}_{j,k}(t)$, and the estimated modal acceleration, $\ddot{y}_{j,k}(t)$.

$$\varepsilon_j(t) = \sum_{k=1}^{NIF} [\ddot{z}_{j,k}(t) - \ddot{y}_{j,k}(t)]^2 \tag{2}$$

where $\varepsilon_j(t)$ is the error function for the jth mode, NIF is the Number of Instrumented Floors, and $\ddot{y}_{j,k}(t)$ is the estimated time variation of the jth mode's relative acceleration at the kth floor. In order to calculate the modal acceleration, $\ddot{z}_{j,k}(t)$, the modal frequencies of the building can first be identified using the Fourier spectrum analysis and second, the recorded accelerations at each instrumented floor are narrow band-pass filtered around each modal frequency of the building in order to calculate the modal accelerations [9]. The summation in the error function (2) is only over the instrumented floors; therefore, the coefficients of $C_{t,j}$ for Timoshenko beam can be estimated by minimizing the error function (2) with respect to the coefficient $C_{t,j}$ as

$$\frac{\partial \varepsilon_j(t)}{\partial C_{t,j}} = 0 \tag{3}$$

which will lead to (4a) and (4b)

$$\frac{\partial \varepsilon_j(t)}{\partial C_{t,j}} = \sum_{k=1}^{NIF} -2\phi_{t,j,k} \cdot \ddot{z}_{j,k}(t) \cdot \ddot{D}_j(t) + 2C_{t,j} \cdot \phi_{t,j,k}^2 \cdot \ddot{D}_j^2(t) \tag{4a}$$

$$\left(\sum_{k=1}^{NIF} \phi_{t,j,k}^2\right) \cdot (C_{t,j} \cdot \ddot{D}_j(t)) = \sum_{k=1}^{NIF} \phi_{t,j,k} \cdot z_{j,k}(t) \tag{4b}$$

where $\ddot{D}_j(t)$ is the jth modal relative floor acceleration (also velocity or displacement) of a multi-story building and is defined as $\ddot{D}_j(t) = \Gamma_j \cdot \ddot{q}_j(t)$ where Γ_j is the jth modal participation factor and $\ddot{q}_j(t)$ is the time-variations of the acceleration of the jth mode of a single-degree-of-freedom system [4]. Eq. (3) can then be simplified for each jth mode as

$$M_j \cdot W_j(t) = Z_j(t) \tag{5}$$

where M_j is a constant time-invariant matrix, and it is a function of the jth mode shapes of a Timoshenko beam; $W_j(t)$ contains the weighting factors for the contributions of the shear beam and the bending beam to the jth modal acceleration at time t; and $Z_j(t)$ is the input matrix containing the calculated modal responses at the instrumented floors at time t. Eq. (5) has to be satisfied at every time step, t. Note that the matrix M_j is time-independent; therefore, it needs to be calculated only once. However, the matrix $Z_j(t)$ is time-dependent and must be calculated at every time step, t. The jth modal acceleration at the kth floor, $\ddot{y}_{j,k}(t)$ can then be calculated by multiplying $W_j(t)$ by $\phi_{t,j,k}$ as

$$\ddot{y}_{j,k}(t) = \phi_{t,j,k} \cdot (C_{t,j} \cdot \ddot{D}_j(t)) \tag{6}$$

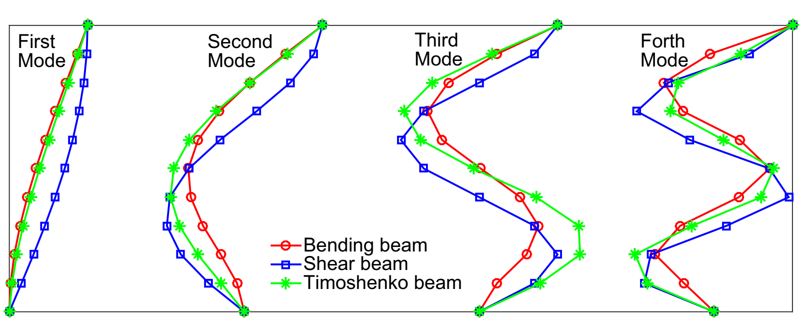

Fig. 1. The first four normalized mode shapes of the inverted bending, shear, and Timoshenko beams. Due to the differences in boundary conditions, all the beam models result in different mode shapes: curvature, amplitude, local maximum along the length of the beam, and the shape of the modes.